

# Mathematica 11.3 Integration Test Results

Test results for the 249 problems in "3.1.5 u (a+b log(c x^n))^p.m"

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \log[c x^n])^3 \log[1 + e x]}{x^2} dx$$

Optimal (type 4, 342 leaves, 14 steps) :

$$\begin{aligned} & 6 b^3 e n^3 \log[x] - 6 b^2 e n^2 \log\left[1 + \frac{1}{e x}\right] (a + b \log[c x^n]) - \\ & 3 b e n \log\left[1 + \frac{1}{e x}\right] (a + b \log[c x^n])^2 - e \log\left[1 + \frac{1}{e x}\right] (a + b \log[c x^n])^3 - \\ & 6 b^3 e n^3 \log[1 + e x] - \frac{6 b^3 n^3 \log[1 + e x]}{x} - \frac{6 b^2 n^2 (a + b \log[c x^n]) \log[1 + e x]}{x} - \\ & \frac{3 b n (a + b \log[c x^n])^2 \log[1 + e x]}{x} - \frac{(a + b \log[c x^n])^3 \log[1 + e x]}{x} + \\ & 6 b^3 e n^3 \text{PolyLog}\left[2, -\frac{1}{e x}\right] + 6 b^2 e n^2 (a + b \log[c x^n]) \text{PolyLog}\left[2, -\frac{1}{e x}\right] + \\ & 3 b e n (a + b \log[c x^n])^2 \text{PolyLog}\left[2, -\frac{1}{e x}\right] + 6 b^3 e n^3 \text{PolyLog}\left[3, -\frac{1}{e x}\right] + \\ & 6 b^2 e n^2 (a + b \log[c x^n]) \text{PolyLog}\left[3, -\frac{1}{e x}\right] + 6 b^3 e n^3 \text{PolyLog}\left[4, -\frac{1}{e x}\right] \end{aligned}$$

Result (type 4, 770 leaves) :

$$\begin{aligned}
& a^3 e \operatorname{Log}[x] + 3 a^2 b e n \operatorname{Log}[x] + 6 a b^2 e n^2 \operatorname{Log}[x] + 6 b^3 e n^3 \operatorname{Log}[x] - \\
& \frac{3}{2} a^2 b e n \operatorname{Log}[x]^2 - 3 a b^2 e n^2 \operatorname{Log}[x]^2 - 3 b^3 e n^3 \operatorname{Log}[x]^2 + a b^2 e n^2 \operatorname{Log}[x]^3 + \\
& b^3 e n^3 \operatorname{Log}[x]^3 - \frac{1}{4} b^3 e n^3 \operatorname{Log}[x]^4 + 3 a^2 b e \operatorname{Log}[x] \operatorname{Log}[c x^n] + 6 a b^2 e n \operatorname{Log}[x] \operatorname{Log}[c x^n] + \\
& 6 b^3 e n^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] - 3 a b^2 e n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] - 3 b^3 e n^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + \\
& b^3 e n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] + 3 a b^2 e \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + 3 b^3 e n \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 - \\
& \frac{3}{2} b^3 e n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 + b^3 e \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 - a^3 e \operatorname{Log}[1 + e x] - 3 a^2 b e n \operatorname{Log}[1 + e x] - \\
& 6 a b^2 e n^2 \operatorname{Log}[1 + e x] - 6 b^3 e n^3 \operatorname{Log}[1 + e x] - \frac{a^3 \operatorname{Log}[1 + e x]}{x} - \frac{3 a^2 b n \operatorname{Log}[1 + e x]}{x} - \\
& \frac{6 a b^2 n^2 \operatorname{Log}[1 + e x]}{x} - \frac{6 b^3 n^3 \operatorname{Log}[1 + e x]}{x} - 3 a^2 b e \operatorname{Log}[c x^n] \operatorname{Log}[1 + e x] - \\
& 6 a b^2 e n \operatorname{Log}[c x^n] \operatorname{Log}[1 + e x] - 6 b^3 e n^2 \operatorname{Log}[c x^n] \operatorname{Log}[1 + e x] - \\
& \frac{3 a^2 b \operatorname{Log}[c x^n] \operatorname{Log}[1 + e x]}{x} - \frac{6 a b^2 n \operatorname{Log}[c x^n] \operatorname{Log}[1 + e x]}{x} - \frac{6 b^3 n^2 \operatorname{Log}[c x^n] \operatorname{Log}[1 + e x]}{x} - \\
& 3 a b^2 e \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + e x] - 3 b^3 e n \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + e x] - \frac{3 a b^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + e x]}{x} - \\
& \frac{3 b^3 n \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + e x]}{x} - b^3 e \operatorname{Log}[c x^n]^3 \operatorname{Log}[1 + e x] - \frac{b^3 \operatorname{Log}[c x^n]^3 \operatorname{Log}[1 + e x]}{x} - \\
& 3 b e n (a^2 + 2 a b n + 2 b^2 n^2 + 2 b (a + b n) \operatorname{Log}[c x^n] + b^2 \operatorname{Log}[c x^n]^2) \operatorname{PolyLog}[2, -e x] + \\
& 6 b^2 e n^2 (a + b n + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -e x] - 6 b^3 e n^3 \operatorname{PolyLog}[4, -e x]
\end{aligned}$$

**Problem 23: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x]}{x^3} dx$$

Optimal (type 4, 470 leaves, 22 steps):

$$\begin{aligned}
& -\frac{45 b^3 e n^3}{8 x} - \frac{3}{8} b^3 e^2 n^3 \log[x] - \frac{21 b^2 e n^2 (a + b \log[c x^n])}{4 x} + \\
& \frac{3}{4} b^2 e^2 n^2 \log[1 + \frac{1}{e x}] (a + b \log[c x^n]) - \frac{9 b e n (a + b \log[c x^n])^2}{4 x} + \\
& \frac{3}{4} b e^2 n \log[1 + \frac{1}{e x}] (a + b \log[c x^n])^2 - \frac{e (a + b \log[c x^n])^3}{2 x} + \frac{1}{2} e^2 \log[1 + \frac{1}{e x}] (a + b \log[c x^n])^3 + \\
& \frac{3}{8} b^3 e^2 n^3 \log[1 + e x] - \frac{3 b^3 n^3 \log[1 + e x]}{8 x^2} - \frac{3 b^2 n^2 (a + b \log[c x^n]) \log[1 + e x]}{4 x^2} - \\
& \frac{3 b n (a + b \log[c x^n])^2 \log[1 + e x]}{4 x^2} - \frac{(a + b \log[c x^n])^3 \log[1 + e x]}{2 x^2} - \\
& \frac{3}{4} b^3 e^2 n^3 \text{PolyLog}[2, -\frac{1}{e x}] - \frac{3}{2} b^2 e^2 n^2 (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{1}{e x}] - \\
& \frac{3}{2} b e^2 n (a + b \log[c x^n])^2 \text{PolyLog}[2, -\frac{1}{e x}] - \frac{3}{2} b^3 e^2 n^3 \text{PolyLog}[3, -\frac{1}{e x}] - \\
& 3 b^2 e^2 n^2 (a + b \log[c x^n]) \text{PolyLog}[3, -\frac{1}{e x}] - 3 b^3 e^2 n^3 \text{PolyLog}[4, -\frac{1}{e x}]
\end{aligned}$$

Result (type 4, 1047 leaves) :

$$\begin{aligned}
& -\frac{1}{8 x^2} \left( 4 a^3 e x + 18 a^2 b e n x + 42 a b^2 e n^2 x + 45 b^3 e n^3 x + 4 a^3 e^2 x^2 \log[x] + \right. \\
& 6 a^2 b e^2 n x^2 \log[x] + 6 a b^2 e^2 n^2 x^2 \log[x] + 3 b^3 e^2 n^3 x^2 \log[x] - 6 a^2 b e^2 n x^2 \log[x]^2 - \\
& 6 a b^2 e^2 n^2 x^2 \log[x]^2 - 3 b^3 e^2 n^3 x^2 \log[x]^2 + 4 a b^2 e^2 n^2 x^2 \log[x]^3 + \\
& 2 b^3 e^2 n^3 x^2 \log[x]^3 - b^3 e^2 n^3 x^2 \log[x]^4 + 12 a^2 b e x \log[c x^n] + 36 a b^2 e n x \log[c x^n] + \\
& 42 b^3 e n^2 x \log[c x^n] + 12 a^2 b e^2 x^2 \log[x] \log[c x^n] + 12 a b^2 e^2 n x^2 \log[x] \log[c x^n] + \\
& 6 b^3 e^2 n^2 x^2 \log[x] \log[c x^n] - 12 a b^2 e^2 n x^2 \log[x]^2 \log[c x^n] - \\
& 6 b^3 e^2 n^2 x^2 \log[x]^2 \log[c x^n] + 4 b^3 e^2 n^2 x^2 \log[x]^3 \log[c x^n] + 12 a b^2 e x \log[c x^n]^2 + \\
& 18 b^3 e n x \log[c x^n]^2 + 12 a b^2 e^2 x^2 \log[x] \log[c x^n]^2 + 6 b^3 e^2 n x^2 \log[x] \log[c x^n]^2 - \\
& 6 b^3 e^2 n x^2 \log[x]^2 \log[c x^n]^2 + 4 b^3 e x \log[c x^n]^3 + 4 b^3 e^2 x^2 \log[x] \log[c x^n]^3 + \\
& 4 a^3 \log[1 + e x] + 6 a^2 b n \log[1 + e x] + 6 a b^2 n^2 \log[1 + e x] + 3 b^3 n^3 \log[1 + e x] - \\
& 4 a^3 e^2 x^2 \log[1 + e x] - 6 a^2 b e^2 n x^2 \log[1 + e x] - 6 a b^2 e^2 n^2 x^2 \log[1 + e x] - \\
& 3 b^3 e^2 n^3 x^2 \log[1 + e x] + 12 a^2 b \log[c x^n] \log[1 + e x] + 12 a b^2 n \log[c x^n] \log[1 + e x] + \\
& 6 b^3 n^2 \log[c x^n] \log[1 + e x] - 12 a^2 b e^2 x^2 \log[c x^n] \log[1 + e x] - \\
& 12 a b^2 e^2 n x^2 \log[c x^n] \log[1 + e x] - 6 b^3 e^2 n^2 x^2 \log[c x^n] \log[1 + e x] + \\
& 12 a b^2 \log[c x^n]^2 \log[1 + e x] + 6 b^3 n \log[c x^n]^2 \log[1 + e x] - \\
& 12 a b^2 e^2 x^2 \log[c x^n]^2 \log[1 + e x] - 6 b^3 e^2 n x^2 \log[c x^n]^2 \log[1 + e x] + \\
& 4 b^3 \log[c x^n]^3 \log[1 + e x] - 4 b^3 e^2 x^2 \log[c x^n]^3 \log[1 + e x] - \\
& 6 b e^2 n x^2 (2 a^2 + 2 a b n + b^2 n^2 + 2 b (2 a + b n) \log[c x^n] + 2 b^2 \log[c x^n]^2) \text{PolyLog}[2, -e x] + \\
& \left. 12 b^2 e^2 n^2 x^2 (2 a + b n + 2 b \log[c x^n]) \text{PolyLog}[3, -e x] - 24 b^3 e^2 n^3 x^2 \text{PolyLog}[4, -e x] \right)
\end{aligned}$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 (a + b \log[c x^n]) \log[d \left(\frac{1}{d} + f x^2\right)] dx$$

Optimal (type 4, 180 leaves, 7 steps) :

$$\begin{aligned}
& -\frac{3 b n x^2}{16 d f} + \frac{1}{16} b n x^4 + \frac{x^2 (a + b \log[c x^n])}{4 d f} - \frac{1}{8} x^4 (a + b \log[c x^n]) + \\
& \frac{b n \log[1 + d f x^2]}{16 d^2 f^2} - \frac{1}{16} b n x^4 \log[1 + d f x^2] - \frac{(a + b \log[c x^n]) \log[1 + d f x^2]}{4 d^2 f^2} + \\
& \frac{1}{4} x^4 (a + b \log[c x^n]) \log[1 + d f x^2] - \frac{b n \text{PolyLog}[2, -d f x^2]}{8 d^2 f^2}
\end{aligned}$$

Result (type 4, 356 leaves) :

$$\begin{aligned}
& \frac{a x^2}{4 d f} - \frac{a x^4}{8} + \frac{1}{32} b x^4 (n - 4 (-n \log[x] + \log[c x^n])) + \frac{b x^2 (-n + 4 (-n \log[x] + \log[c x^n]))}{16 d f} - \\
& \frac{a \log[1 + d f x^2]}{4 d^2 f^2} + \frac{1}{4} a x^4 \log[1 + d f x^2] + \frac{b (n - 4 (-n \log[x] + \log[c x^n])) \log[1 + d f x^2]}{16 d^2 f^2} + \\
& \frac{1}{16} b x^4 (-n + 4 n \log[x] + 4 (-n \log[x] + \log[c x^n])) \log[1 + d f x^2] - \\
& \frac{1}{2} b d f n \left( -\frac{-\frac{x^2}{4} + \frac{1}{2} x^2 \log[x]}{d^2 f^2} + \frac{-\frac{x^4}{16} + \frac{1}{4} x^4 \log[x]}{d f} + \right. \\
& \left. \frac{1}{d^2 f^2} \left( \frac{\log[x] \log[1 + \pm \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, -\pm \sqrt{d} \sqrt{f} x]}{2 d f} + \right. \right. \\
& \left. \left. \frac{\log[x] \log[1 - \pm \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, \pm \sqrt{d} \sqrt{f} x]}{2 d f} \right) \right)
\end{aligned}$$

Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x (a + b \log[c x^n]) \log[d \left(\frac{1}{d} + f x^2\right)] dx$$

Optimal (type 4, 114 leaves, 8 steps) :

$$\begin{aligned}
& \frac{1}{2} b n x^2 - \frac{1}{2} x^2 (a + b \log[c x^n]) - \frac{b n (1 + d f x^2) \log[1 + d f x^2]}{4 d f} + \\
& \frac{(1 + d f x^2) (a + b \log[c x^n]) \log[1 + d f x^2]}{2 d f} + \frac{b n \text{PolyLog}[2, -d f x^2]}{4 d f}
\end{aligned}$$

Result (type 4, 286 leaves) :

$$\begin{aligned}
& -\frac{ax^2}{2} + \frac{1}{4}bx^2(n - 2(-n \operatorname{Log}[x] + \operatorname{Log}[cx^n])) + \frac{a \operatorname{Log}[1 + dfx^2]}{2df} + \\
& \frac{1}{2}a x^2 \operatorname{Log}[1 + dfx^2] + \frac{b(-n + 2(-n \operatorname{Log}[x] + \operatorname{Log}[cx^n])) \operatorname{Log}[1 + dfx^2]}{4df} + \\
& \frac{1}{4}b x^2 (-n + 2n \operatorname{Log}[x] + 2(-n \operatorname{Log}[x] + \operatorname{Log}[cx^n])) \operatorname{Log}[1 + dfx^2] - \\
& b d f n \left( \frac{-\frac{x^2}{4} + \frac{1}{2}x^2 \operatorname{Log}[x]}{df} - \frac{1}{df} \left( \frac{\operatorname{Log}[x] \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x]}{2df} + \right. \right. \\
& \left. \left. \frac{\operatorname{Log}[x] \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x]}{2df} \right) \right)
\end{aligned}$$

**Problem 26:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[cx^n]) \operatorname{Log}[d(\frac{1}{d} + fx^2)]}{x} dx$$

Optimal (type 4, 39 leaves, 2 steps):

$$-\frac{1}{2}(a + b \operatorname{Log}[cx^n]) \operatorname{PolyLog}[2, -dfx^2] + \frac{1}{4}bn \operatorname{PolyLog}[3, -dfx^2]$$

Result (type 4, 319 leaves):

$$\begin{aligned}
& \frac{1}{2}b \operatorname{Log}[x] (n \operatorname{Log}[x] + 2(-n \operatorname{Log}[x] + \operatorname{Log}[cx^n])) \operatorname{Log}[1 + dfx^2] - \\
& 2bd f (-n \operatorname{Log}[x] + \operatorname{Log}[cx^n]) \left( \frac{\operatorname{Log}[x] \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x]}{2df} + \right. \\
& \left. \frac{\operatorname{Log}[x] \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x]}{2df} \right) - \\
& \frac{1}{2}a \operatorname{PolyLog}[2, -dfx^2] - bd f n \left( \frac{1}{df} \left( \frac{1}{2} \operatorname{Log}[x]^2 \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + \right. \right. \\
& \left. \left. \operatorname{Log}[x] \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] - \operatorname{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] \right) + \frac{1}{df} \right. \\
& \left. \left( \frac{1}{2} \operatorname{Log}[x]^2 \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \operatorname{Log}[x] \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - \operatorname{PolyLog}[3, i \sqrt{d} \sqrt{f} x] \right) \right)
\end{aligned}$$

**Problem 27:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{Log}[cx^n]) \operatorname{Log}[d(\frac{1}{d} + fx^2)]}{x^3} dx$$

Optimal (type 4, 141 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{2} b d f n \log[x] - \frac{1}{2} b d f n \log[x]^2 + d f \log[x] (a + b \log[c x^n]) - \\ & \frac{1}{4} b d f n \log[1 + d f x^2] - \frac{b n \log[1 + d f x^2]}{4 x^2} - \frac{1}{2} d f (a + b \log[c x^n]) \log[1 + d f x^2] - \\ & \frac{(a + b \log[c x^n]) \log[1 + d f x^2]}{2 x^2} - \frac{1}{4} b d f n \text{PolyLog}[2, -d f x^2] \end{aligned}$$

Result (type 4, 252 leaves):

$$\begin{aligned} & a d f \log[x] + \frac{1}{2} b d f \log[x] (n + 2 (-n \log[x] + \log[c x^n])) - \frac{1}{2} a d f \log[1 + d f x^2] - \\ & \frac{a \log[1 + d f x^2]}{2 x^2} - \frac{1}{4} b d f (n + 2 (-n \log[x] + \log[c x^n])) \log[1 + d f x^2] - \\ & \frac{b (n + 2 n \log[x] + 2 (-n \log[x] + \log[c x^n])) \log[1 + d f x^2]}{4 x^2} + \\ & b d f n \left( \frac{\log[x]^2}{2} - d f \left( \frac{\log[x] \log[1 + i \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x]}{2 d f} + \right. \right. \\ & \left. \left. \frac{\log[x] \log[1 - i \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x]}{2 d f} \right) \right) \end{aligned}$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 (a + b \log[c x^n])^2 \log[d (1/d + f x^2)] dx$$

Optimal (type 4, 367 leaves, 13 steps):

$$\begin{aligned} & \frac{7 b^2 n^2 x^2}{32 d f} - \frac{3}{64} b^2 n^2 x^4 - \frac{3 b n x^2 (a + b \log[c x^n])}{8 d f} + \frac{1}{8} b n x^4 (a + b \log[c x^n]) + \\ & \frac{x^2 (a + b \log[c x^n])^2}{4 d f} - \frac{1}{8} x^4 (a + b \log[c x^n])^2 - \frac{b^2 n^2 \log[1 + d f x^2]}{32 d^2 f^2} + \frac{1}{32} b^2 n^2 x^4 \log[1 + d f x^2] + \\ & \frac{b n (a + b \log[c x^n]) \log[1 + d f x^2]}{8 d^2 f^2} - \frac{1}{8} b n x^4 (a + b \log[c x^n]) \log[1 + d f x^2] - \\ & \frac{(a + b \log[c x^n])^2 \log[1 + d f x^2]}{4 d^2 f^2} + \frac{1}{4} x^4 (a + b \log[c x^n])^2 \log[1 + d f x^2] + \\ & \frac{b^2 n^2 \text{PolyLog}[2, -d f x^2]}{16 d^2 f^2} - \frac{b n (a + b \log[c x^n]) \text{PolyLog}[2, -d f x^2]}{4 d^2 f^2} + \frac{b^2 n^2 \text{PolyLog}[3, -d f x^2]}{8 d^2 f^2} \end{aligned}$$

Result (type 4, 673 leaves):

$$\frac{1}{64 d^2 f^2} \left( 2 d f x^2 \left( 8 a^2 - 4 a b n + b^2 n^2 + 4 b^2 n \left( n \text{Log}[x] - \text{Log}[c x^n] \right) + 16 a b \left( -n \text{Log}[x] + \text{Log}[c x^n] \right) + 8 b^2 \left( -n \text{Log}[x] + \text{Log}[c x^n] \right)^2 \right) - d^2 f^2 x^4 \left( 8 a^2 - 4 a b n + b^2 n^2 + 4 b^2 n \left( n \text{Log}[x] - \text{Log}[c x^n] \right) + 16 a b \left( -n \text{Log}[x] + \text{Log}[c x^n] \right) + 8 b^2 \left( -n \text{Log}[x] + \text{Log}[c x^n] \right)^2 \right) + 2 d^2 f^2 x^4 \left( 8 a^2 - 4 a b n + b^2 n^2 - 4 b \left( -4 a + b n \right) \text{Log}[c x^n] + 8 b^2 \text{Log}[c x^n]^2 \right) \text{Log}[1 + d f x^2] - 2 \left( 8 a^2 - 4 a b n + b^2 n^2 + 4 b^2 n \left( n \text{Log}[x] - \text{Log}[c x^n] \right) + 16 a b \left( -n \text{Log}[x] + \text{Log}[c x^n] \right) + 8 b^2 \left( -n \text{Log}[x] + \text{Log}[c x^n] \right)^2 \right) \text{Log}[1 + d f x^2] + b n \left( -4 a + b n + 4 b n \text{Log}[x] - 4 b \text{Log}[c x^n] \right) \left( 4 d f x^2 - d^2 f^2 x^4 - 8 d f x^2 \text{Log}[x] + 4 d^2 f^2 x^4 \text{Log}[x] + 8 \text{Log}[x] \text{Log}[1 - i \sqrt{d} \sqrt{f} x] + 8 \text{Log}[x] \text{Log}[1 + i \sqrt{d} \sqrt{f} x] + 8 \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 8 \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] \right) - b^2 n^2 \left( -8 d f x^2 + d^2 f^2 x^4 + 16 d f x^2 \text{Log}[x] - 4 d^2 f^2 x^4 \text{Log}[x] - 16 d f x^2 \text{Log}[x]^2 + 8 d^2 f^2 x^4 \text{Log}[x]^2 + 16 \text{Log}[x]^2 \text{Log}[1 - i \sqrt{d} \sqrt{f} x] + 16 \text{Log}[x]^2 \text{Log}[1 + i \sqrt{d} \sqrt{f} x] + 32 \text{Log}[x] \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 32 \text{Log}[x] \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - 32 \text{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 32 \text{PolyLog}[3, i \sqrt{d} \sqrt{f} x] \right)$$

**Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int x \left( a + b \text{Log}[c x^n] \right)^2 \text{Log}[d \left( \frac{1}{d} + f x^2 \right)] dx$$

Optimal (type 4, 241 leaves, 15 steps):

$$-\frac{3}{4} b^2 n^2 x^2 + b n x^2 \left( a + b \text{Log}[c x^n] \right) - \frac{1}{2} x^2 \left( a + b \text{Log}[c x^n] \right)^2 + \frac{b^2 n^2 (1 + d f x^2) \text{Log}[1 + d f x^2]}{4 d f} - \frac{b n (1 + d f x^2) (a + b \text{Log}[c x^n]) \text{Log}[1 + d f x^2]}{2 d f} + \frac{(1 + d f x^2) (a + b \text{Log}[c x^n])^2 \text{Log}[1 + d f x^2]}{2 d f} - \frac{b^2 n^2 \text{PolyLog}[2, -d f x^2]}{4 d f} + \frac{b n (a + b \text{Log}[c x^n]) \text{PolyLog}[2, -d f x^2]}{2 d f} - \frac{b^2 n^2 \text{PolyLog}[3, -d f x^2]}{4 d f}$$

Result (type 4, 519 leaves):

$$\begin{aligned}
& \frac{1}{4 d f} \left( -d f x^2 \left( 2 a^2 - 2 a b n + b^2 n^2 + 2 b^2 n \left( n \log[x] - \log[c x^n] \right) + \right. \right. \\
& \quad 4 a b \left( -n \log[x] + \log[c x^n] \right) + 2 b^2 \left( -n \log[x] + \log[c x^n] \right)^2 \left. \right) + \\
& \quad d f x^2 \left( 2 a^2 - 2 a b n + b^2 n^2 - 2 b \left( -2 a + b n \right) \log[c x^n] + 2 b^2 \log[c x^n]^2 \right) \log[1 + d f x^2] + \\
& \quad \left( 2 a^2 - 2 a b n + b^2 n^2 + 2 b^2 n \left( n \log[x] - \log[c x^n] \right) + 4 a b \left( -n \log[x] + \log[c x^n] \right) + \right. \\
& \quad \left. \left. 2 b^2 \left( -n \log[x] + \log[c x^n] \right)^2 \right) \log[1 + d f x^2] + 2 b n \left( 2 a - b n - 2 b n \log[x] + 2 b \log[c x^n] \right) \right. \\
& \quad \left( \frac{1}{2} d f x^2 - d f x^2 \log[x] + \log[x] \log[1 - i \sqrt{d} \sqrt{f} x] + \log[x] \log[1 + i \sqrt{d} \sqrt{f} x] + \right. \\
& \quad \left. \left. \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] \right) - \right. \\
& \quad b^2 n^2 \left( d f x^2 - 2 d f x^2 \log[x] + 2 d f x^2 \log[x]^2 - 2 \log[x]^2 \log[1 - i \sqrt{d} \sqrt{f} x] - \right. \\
& \quad \left. 2 \log[x]^2 \log[1 + i \sqrt{d} \sqrt{f} x] - 4 \log[x] \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] - 4 \log[x] \right. \\
& \quad \left. \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] + 4 \text{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] + 4 \text{PolyLog}[3, i \sqrt{d} \sqrt{f} x] \right)
\end{aligned}$$

**Problem 34:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \log[c x^n])^2 \log[d \left(\frac{1}{d} + f x^2\right)]}{x} dx$$

Optimal (type 4, 70 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{2} (a + b \log[c x^n])^2 \text{PolyLog}[2, -d f x^2] + \\
& \frac{1}{2} b n (a + b \log[c x^n]) \text{PolyLog}[3, -d f x^2] - \frac{1}{4} b^2 n^2 \text{PolyLog}[4, -d f x^2]
\end{aligned}$$

Result (type 4, 484 leaves):

$$\begin{aligned}
& \frac{1}{3} \left( \log[x] \left( b^2 n^2 \log[x]^2 - 3 b n \log[x] (a + b \log[c x^n]) + 3 (a + b \log[c x^n])^2 \right) \log[1 + d f x^2] - \right. \\
& \quad 3 (a - b n \log[x] + b \log[c x^n])^2 \left( \log[x] \left( \log[1 - i \sqrt{d} \sqrt{f} x] + \log[1 + i \sqrt{d} \sqrt{f} x] \right) + \right. \\
& \quad \left. \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] \right) + 3 b n \\
& \quad (-a + b n \log[x] - b \log[c x^n]) \left( \log[x]^2 \log[1 - i \sqrt{d} \sqrt{f} x] + \log[x]^2 \log[1 + i \sqrt{d} \sqrt{f} x] + \right. \\
& \quad 2 \log[x] \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 2 \log[x] \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - \\
& \quad \left. 2 \text{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 2 \text{PolyLog}[3, i \sqrt{d} \sqrt{f} x] \right) - \\
& b^2 n^2 \left( \log[x]^3 \log[1 - i \sqrt{d} \sqrt{f} x] + \log[x]^3 \log[1 + i \sqrt{d} \sqrt{f} x] + \right. \\
& \quad 3 \log[x]^2 \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 3 \log[x]^2 \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - \\
& \quad 6 \log[x] \text{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 6 \log[x] \text{PolyLog}[3, i \sqrt{d} \sqrt{f} x] + \\
& \quad \left. 6 \text{PolyLog}[4, -i \sqrt{d} \sqrt{f} x] + 6 \text{PolyLog}[4, i \sqrt{d} \sqrt{f} x] \right)
\end{aligned}$$

### Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d \left(\frac{1}{d} + f x^2\right)]}{x^3} dx$$

Optimal (type 4, 257 leaves, 11 steps):

$$\begin{aligned} & \frac{1}{2} b^2 d f n^2 \operatorname{Log}[x] - \frac{1}{2} b d f n \operatorname{Log}\left[1 + \frac{1}{d f x^2}\right] (a + b \operatorname{Log}[c x^n]) - \\ & \frac{1}{2} d f \operatorname{Log}\left[1 + \frac{1}{d f x^2}\right] (a + b \operatorname{Log}[c x^n])^2 - \frac{1}{4} b^2 d f n^2 \operatorname{Log}[1 + d f x^2] - \\ & \frac{b^2 n^2 \operatorname{Log}[1 + d f x^2]}{4 x^2} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + d f x^2]}{2 x^2} - \\ & \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + d f x^2]}{2 x^2} + \frac{1}{4} b^2 d f n^2 \operatorname{PolyLog}[2, -\frac{1}{d f x^2}] + \\ & \frac{1}{2} b d f n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{1}{d f x^2}] + \frac{1}{4} b^2 d f n^2 \operatorname{PolyLog}[3, -\frac{1}{d f x^2}] \end{aligned}$$

Result (type 4, 488 leaves):

$$\begin{aligned} & \frac{1}{4} \left( 2 d f \operatorname{Log}[x] (2 a^2 + 2 a b n + b^2 n^2 + 4 a b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) + \right. \\ & 2 b^2 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 2 b^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 \Big) - \frac{1}{x^2} \\ & (2 a^2 + 2 a b n + b^2 n^2 + 2 b (2 a + b n) \operatorname{Log}[c x^n] + 2 b^2 \operatorname{Log}[c x^n]^2) \operatorname{Log}[1 + d f x^2] - \\ & d f (2 a^2 + 2 a b n + b^2 n^2 + 4 a b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) + \\ & 2 b^2 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 2 b^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 \Big) \\ & \operatorname{Log}[1 + d f x^2] - 2 b d f n (-2 a - b n + 2 b n \operatorname{Log}[x] - 2 b \operatorname{Log}[c x^n]) \\ & (\operatorname{Log}[x] (\operatorname{Log}[x] - \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] - \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x]) - \\ & \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] - \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x]) + \\ & \frac{2}{3} b^2 d f n^2 (2 \operatorname{Log}[x]^3 - 3 \operatorname{Log}[x]^2 \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] - 3 \operatorname{Log}[x]^2 \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] - \\ & 6 \operatorname{Log}[x] \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] - 6 \operatorname{Log}[x] \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] + \\ & \left. 6 \operatorname{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] + 6 \operatorname{PolyLog}[3, i \sqrt{d} \sqrt{f} x] \right) \end{aligned}$$

### Problem 40: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x^3 (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d \left(\frac{1}{d} + f x^2\right)] dx$$

Optimal (type 4, 591 leaves, 22 steps):

$$\begin{aligned}
& -\frac{45 b^3 n^3 x^2}{128 d f} + \frac{3}{64} b^3 n^3 x^4 + \frac{21 b^2 n^2 x^2 (a + b \log[c x^n])}{32 d f} - \frac{9}{64} b^2 n^2 x^4 (a + b \log[c x^n]) - \\
& \frac{9 b n x^2 (a + b \log[c x^n])^2}{16 d f} + \frac{3}{16} b n x^4 (a + b \log[c x^n])^2 + \frac{x^2 (a + b \log[c x^n])^3}{4 d f} - \\
& \frac{1}{8} x^4 (a + b \log[c x^n])^3 + \frac{3 b^3 n^3 \log[1 + d f x^2]}{128 d^2 f^2} - \frac{3}{128} b^3 n^3 x^4 \log[1 + d f x^2] - \\
& \frac{3 b^2 n^2 (a + b \log[c x^n]) \log[1 + d f x^2]}{32 d^2 f^2} + \frac{3}{32} b^2 n^2 x^4 (a + b \log[c x^n]) \log[1 + d f x^2] + \\
& \frac{3 b n (a + b \log[c x^n])^2 \log[1 + d f x^2]}{16 d^2 f^2} - \frac{3}{16} b n x^4 (a + b \log[c x^n])^2 \log[1 + d f x^2] - \\
& \frac{(a + b \log[c x^n])^3 \log[1 + d f x^2]}{4 d^2 f^2} + \frac{1}{4} x^4 (a + b \log[c x^n])^3 \log[1 + d f x^2] - \\
& \frac{3 b^3 n^3 \text{PolyLog}[2, -d f x^2]}{64 d^2 f^2} + \frac{3 b^2 n^2 (a + b \log[c x^n]) \text{PolyLog}[2, -d f x^2]}{16 d^2 f^2} - \\
& \frac{3 b n (a + b \log[c x^n])^2 \text{PolyLog}[2, -d f x^2]}{8 d^2 f^2} - \frac{3 b^3 n^3 \text{PolyLog}[3, -d f x^2]}{32 d^2 f^2} + \\
& \frac{3 b^2 n^2 (a + b \log[c x^n]) \text{PolyLog}[3, -d f x^2]}{8 d^2 f^2} - \frac{3 b^3 n^3 \text{PolyLog}[4, -d f x^2]}{16 d^2 f^2}
\end{aligned}$$

Result (type 4, 1250 leaves) :

$$\begin{aligned}
& \frac{1}{256 d^2 f^2} \\
& \left( 2 d f x^2 \left( 32 a^3 - 24 a^2 b n + 12 a b^2 n^2 - 3 b^3 n^3 + 48 a b^2 n (\ln \log[x] - \log[c x^n]) + 96 a^2 b (-n \ln \log[x] + \log[c x^n]) + 12 b^3 n^2 (-n \ln \log[x] + \log[c x^n]) + 96 a b^2 (-n \ln \log[x] + \log[c x^n])^2 - 24 b^3 n (-n \ln \log[x] + \log[c x^n])^2 + 32 b^3 (-n \ln \log[x] + \log[c x^n])^3 \right) - \right. \\
& d^2 f^2 x^4 \left( 32 a^3 - 24 a^2 b n + 12 a b^2 n^2 - 3 b^3 n^3 + 48 a b^2 n (\ln \log[x] - \log[c x^n]) + 96 a^2 b (-n \ln \log[x] + \log[c x^n]) + 12 b^3 n^2 (-n \ln \log[x] + \log[c x^n]) + 96 a b^2 (-n \ln \log[x] + \log[c x^n])^2 - 24 b^3 n (-n \ln \log[x] + \log[c x^n])^2 + 32 b^3 (-n \ln \log[x] + \log[c x^n])^3 \right) + \\
& 2 d^2 f^2 x^4 \left( 32 a^3 - 24 a^2 b n + 12 a b^2 n^2 - 3 b^3 n^3 + 12 b (8 a^2 - 4 a b n + b^2 n^2) \ln \log[c x^n] - 24 b^2 (-4 a + b n) \ln \log[c x^n]^2 + 32 b^3 \ln \log[c x^n]^3 \right) \ln \log[1 + d f x^2] - \\
& 2 \left( 32 a^3 - 24 a^2 b n + 12 a b^2 n^2 - 3 b^3 n^3 + 48 a b^2 n (\ln \log[x] - \log[c x^n]) + 96 a^2 b (-n \ln \log[x] + \log[c x^n]) + 12 b^3 n^2 (-n \ln \log[x] + \log[c x^n]) + 96 a b^2 (-n \ln \log[x] + \log[c x^n])^2 - 24 b^3 n (-n \ln \log[x] + \log[c x^n])^2 + 32 b^3 (-n \ln \log[x] + \log[c x^n])^3 \right) \ln \log[1 + d f x^2] - \\
& 24 b n \left( 8 a^2 - 4 a b n + b^2 n^2 + 4 b^2 n (\ln \log[x] - \log[c x^n]) + 16 a b (-n \ln \log[x] + \log[c x^n]) + 8 b^2 (-n \ln \log[x] + \log[c x^n])^2 \right) \\
& \left( \frac{1}{2} d f x^2 - \frac{1}{8} d^2 f^2 x^4 - d f x^2 \ln \log[x] + \frac{1}{2} d^2 f^2 x^4 \ln \log[x] + \ln \log[x] \ln \log[1 - i \sqrt{d} \sqrt{f} x] + \ln \log[x] \ln \log[1 + i \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] \right) + \\
& 3 b^2 n^2 \left( -4 a + b n + 4 b n \ln \log[x] - 4 b \ln \log[c x^n] \right) \\
& \left( -8 d f x^2 + d^2 f^2 x^4 + 16 d f x^2 \ln \log[x] - 4 d^2 f^2 x^4 \ln \log[x] - 16 d f x^2 \ln \log[x]^2 + 8 d^2 f^2 x^4 \ln \log[x]^2 + 16 \ln \log[x]^2 \ln \log[1 - i \sqrt{d} \sqrt{f} x] + 16 \ln \log[x]^2 \ln \log[1 + i \sqrt{d} \sqrt{f} x] + 32 \ln \log[x] \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 32 \ln \log[x] \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - 32 \text{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 32 \text{PolyLog}[3, i \sqrt{d} \sqrt{f} x] \right) + \\
& b^3 n^3 \left( d^2 f^2 x^4 (3 - 12 \ln \log[x] + 24 \ln \log[x]^2 - 32 \ln \log[x]^3) + 16 d f x^2 (-3 + 6 \ln \log[x] - 6 \ln \log[x]^2 + 4 \ln \log[x]^3) - 64 \left( \ln \log[x]^3 \ln \log[1 - i \sqrt{d} \sqrt{f} x] + \ln \log[x]^3 \ln \log[1 + i \sqrt{d} \sqrt{f} x] + 3 \ln \log[x]^2 \text{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 3 \ln \log[x]^2 \text{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - 6 \ln \log[x] \text{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 6 \ln \log[x] \text{PolyLog}[3, i \sqrt{d} \sqrt{f} x] + 6 \text{PolyLog}[4, -i \sqrt{d} \sqrt{f} x] + 6 \text{PolyLog}[4, i \sqrt{d} \sqrt{f} x] \right) \right)
\end{aligned}$$

**Problem 41:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x (a + b \ln \log[c x^n])^3 \ln \log[d (1/d + f x^2)] dx$$

Optimal (type 4, 411 leaves, 24 steps):

$$\begin{aligned}
& \frac{3}{2} b^3 n^3 x^2 - \frac{9}{4} b^2 n^2 x^2 (a + b \operatorname{Log}[c x^n]) + \frac{3}{2} b n x^2 (a + b \operatorname{Log}[c x^n])^2 - \frac{1}{2} x^2 (a + b \operatorname{Log}[c x^n])^3 - \\
& \frac{3 b^3 n^3 (1 + d f x^2) \operatorname{Log}[1 + d f x^2]}{8 d f} + \frac{3 b^2 n^2 (1 + d f x^2) (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + d f x^2]}{4 d f} - \\
& \frac{3 b n (1 + d f x^2) (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + d f x^2]}{4 d f} + \frac{(1 + d f x^2) (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + d f x^2]}{2 d f} + \\
& \frac{3 b^3 n^3 \operatorname{PolyLog}[2, -d f x^2]}{8 d f} - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -d f x^2]}{4 d f} + \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -d f x^2]}{4 d f} + \frac{3 b^3 n^3 \operatorname{PolyLog}[3, -d f x^2]}{8 d f} - \\
& \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -d f x^2]}{4 d f} + \frac{3 b^3 n^3 \operatorname{PolyLog}[4, -d f x^2]}{8 d f}
\end{aligned}$$

Result (type 4, 990 leaves):

$$\begin{aligned}
& \frac{1}{8 d f} \left( -d f x^2 (4 a^3 - 6 a^2 b n + 6 a b^2 n^2 - 3 b^3 n^3 + 12 a b^2 n (\operatorname{n Log}[x] - \operatorname{Log}[c x^n])) + \right. \\
& 12 a^2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 6 b^3 n^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 12 a b^2 \\
& (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 - 6 b^3 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 + 4 b^3 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^3 + \\
& d f x^2 (4 a^3 - 6 a^2 b n + 6 a b^2 n^2 - 3 b^3 n^3 + 6 b (2 a^2 - 2 a b n + b^2 n^2) \operatorname{Log}[c x^n] - \\
& 6 b^2 (-2 a + b n) \operatorname{Log}[c x^n]^2 + 4 b^3 \operatorname{Log}[c x^n]^3) \operatorname{Log}[1 + d f x^2] + \\
& (4 a^3 - 6 a^2 b n + 6 a b^2 n^2 - 3 b^3 n^3 + 12 a b^2 n (\operatorname{n Log}[x] - \operatorname{Log}[c x^n])) + 12 a^2 b \\
& (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 6 b^3 n^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 12 a b^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 - \\
& 6 b^3 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 + 4 b^3 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^3) \operatorname{Log}[1 + d f x^2] + \\
& 6 b n (2 a^2 - 2 a b n + b^2 n^2 + 2 b^2 n (\operatorname{n Log}[x] - \operatorname{Log}[c x^n])) + 4 a b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + \\
& 2 b^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 \left( \frac{1}{2} d f x^2 - d f x^2 \operatorname{Log}[x] + \operatorname{Log}[x] \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \right. \\
& \operatorname{Log}[x] \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] \Big) + \\
& 3 b^2 n^2 (-2 a + b n + 2 b n \operatorname{Log}[x] - 2 b \operatorname{Log}[c x^n]) \left( d f x^2 - 2 d f x^2 \operatorname{Log}[x] + \right. \\
& 2 d f x^2 \operatorname{Log}[x]^2 - 2 \operatorname{Log}[x]^2 \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] - 2 \operatorname{Log}[x]^2 \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] - \\
& 4 \operatorname{Log}[x] \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] - 4 \operatorname{Log}[x] \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] + \\
& 4 \operatorname{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] + 4 \operatorname{PolyLog}[3, i \sqrt{d} \sqrt{f} x] \Big) + \\
& b^3 n^3 \left( d f x^2 (3 - 6 \operatorname{Log}[x] + 6 \operatorname{Log}[x]^2 - 4 \operatorname{Log}[x]^3) + 4 \left( \operatorname{Log}[x]^3 \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \right. \right. \\
& \operatorname{Log}[x]^3 \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + 3 \operatorname{Log}[x]^2 \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + \\
& 3 \operatorname{Log}[x]^2 \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - 6 \operatorname{Log}[x] \operatorname{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 6 \operatorname{Log}[x] \\
& \operatorname{PolyLog}[3, i \sqrt{d} \sqrt{f} x] + 6 \operatorname{PolyLog}[4, -i \sqrt{d} \sqrt{f} x] + 6 \operatorname{PolyLog}[4, i \sqrt{d} \sqrt{f} x] \Big) \Big)
\end{aligned}$$

**Problem 42:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (\frac{1}{d} + f x^2)]}{x} dx$$

Optimal (type 4, 101 leaves, 4 steps) :

$$\begin{aligned} & -\frac{1}{2} (a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}[2, -d f x^2] + \frac{3}{4} b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[3, -d f x^2] - \\ & \frac{3}{4} b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[4, -d f x^2] + \frac{3}{8} b^3 n^3 \operatorname{PolyLog}[5, -d f x^2] \end{aligned}$$

Result (type 4, 754 leaves) :

$$\begin{aligned} & \frac{1}{4} (-\operatorname{Log}[x] (b^3 n^3 \operatorname{Log}[x]^3 - 4 b^2 n^2 \operatorname{Log}[x]^2 (a + b \operatorname{Log}[c x^n])) + \\ & 6 b n \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])^2 - 4 (a + b \operatorname{Log}[c x^n])^3) \operatorname{Log}[1 + d f x^2] - \\ & 4 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3 (\operatorname{Log}[x] (\operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x]) + \\ & \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x]) - 6 b n \\ & (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 (\operatorname{Log}[x]^2 \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \operatorname{Log}[x]^2 \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + \\ & 2 \operatorname{Log}[x] \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 2 \operatorname{Log}[x] \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - \\ & 2 \operatorname{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 2 \operatorname{PolyLog}[3, i \sqrt{d} \sqrt{f} x]) + 4 b^2 n^2 \\ & (-a + b n \operatorname{Log}[x] - b \operatorname{Log}[c x^n]) (\operatorname{Log}[x]^3 \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \operatorname{Log}[x]^3 \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + \\ & 3 \operatorname{Log}[x]^2 \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 3 \operatorname{Log}[x]^2 \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - \\ & 6 \operatorname{Log}[x] \operatorname{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 6 \operatorname{Log}[x] \operatorname{PolyLog}[3, i \sqrt{d} \sqrt{f} x] + \\ & 6 \operatorname{PolyLog}[4, -i \sqrt{d} \sqrt{f} x] + 6 \operatorname{PolyLog}[4, i \sqrt{d} \sqrt{f} x]) - \\ & b^3 n^3 (\operatorname{Log}[x]^4 \operatorname{Log}[1 - i \sqrt{d} \sqrt{f} x] + \operatorname{Log}[x]^4 \operatorname{Log}[1 + i \sqrt{d} \sqrt{f} x] + \\ & 4 \operatorname{Log}[x]^3 \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] + 4 \operatorname{Log}[x]^3 \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] - \\ & 12 \operatorname{Log}[x]^2 \operatorname{PolyLog}[3, -i \sqrt{d} \sqrt{f} x] - 12 \operatorname{Log}[x]^2 \operatorname{PolyLog}[3, i \sqrt{d} \sqrt{f} x] + \\ & 24 \operatorname{Log}[x] \operatorname{PolyLog}[4, -i \sqrt{d} \sqrt{f} x] + 24 \operatorname{Log}[x] \operatorname{PolyLog}[4, i \sqrt{d} \sqrt{f} x] - \\ & 24 \operatorname{PolyLog}[5, -i \sqrt{d} \sqrt{f} x] - 24 \operatorname{PolyLog}[5, i \sqrt{d} \sqrt{f} x]) \end{aligned}$$

**Problem 43:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (\frac{1}{d} + f x^2)]}{x^3} dx$$

Optimal (type 4, 425 leaves, 15 steps) :

$$\begin{aligned}
& \frac{3}{4} b^3 d f n^3 \operatorname{Log}[x] - \frac{3}{4} b^2 d f n^2 \operatorname{Log}\left[1 + \frac{1}{d f x^2}\right] (a + b \operatorname{Log}[c x^n]) - \\
& \frac{3}{4} b d f n \operatorname{Log}\left[1 + \frac{1}{d f x^2}\right] (a + b \operatorname{Log}[c x^n])^2 - \frac{1}{2} d f \operatorname{Log}\left[1 + \frac{1}{d f x^2}\right] (a + b \operatorname{Log}[c x^n])^3 - \\
& \frac{3}{8} b^3 d f n^3 \operatorname{Log}[1 + d f x^2] - \frac{3 b^3 n^3 \operatorname{Log}[1 + d f x^2]}{8 x^2} - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + d f x^2]}{4 x^2} - \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + d f x^2]}{4 x^2} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + d f x^2]}{2 x^2} + \\
& \frac{3}{8} b^3 d f n^3 \operatorname{PolyLog}[2, -\frac{1}{d f x^2}] + \frac{3}{4} b^2 d f n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{1}{d f x^2}] + \\
& \frac{3}{4} b d f n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{1}{d f x^2}] + \frac{3}{8} b^3 d f n^3 \operatorname{PolyLog}[3, -\frac{1}{d f x^2}] + \\
& \frac{3}{4} b^2 d f n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{1}{d f x^2}] + \frac{3}{8} b^3 d f n^3 \operatorname{PolyLog}[4, -\frac{1}{d f x^2}]
\end{aligned}$$

Result (type 4, 940 leaves):

$$\begin{aligned}
& \frac{1}{8} \left( 2 d f \operatorname{Log}[x] (4 a^3 + 6 a^2 b n + 6 a b^2 n^2 + 3 b^3 n^3 + 12 a^2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + \right. \\
& \quad 12 a b^2 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 6 b^3 n^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 12 a b^2 \\
& \quad (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 + 6 b^3 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 + 4 b^3 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^3) - \\
& \quad \frac{1}{x^2} (4 a^3 + 6 a^2 b n + 6 a b^2 n^2 + 3 b^3 n^3 + 6 b (2 a^2 + 2 a b n + b^2 n^2) \operatorname{Log}[c x^n] + \\
& \quad 6 b^2 (2 a + b n) \operatorname{Log}[c x^n]^2 + 4 b^3 \operatorname{Log}[c x^n]^3) \operatorname{Log}[1 + d f x^2] - \\
& \quad d f (4 a^3 + 6 a^2 b n + 6 a b^2 n^2 + 3 b^3 n^3 + 12 a^2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 12 a b^2 n \\
& \quad (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 6 b^3 n^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 12 a b^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 + \\
& \quad 6 b^3 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 + 4 b^3 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^3) \operatorname{Log}[1 + d f x^2] + \\
& \quad 6 b d f n (2 a^2 + 2 a b n + b^2 n^2 + 4 a b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) + \\
& \quad 2 b^2 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + 2 b^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 \\
& \quad (\operatorname{Log}[x] (\operatorname{Log}[x] - \operatorname{Log}[1 - \frac{1}{2} \sqrt{d} \sqrt{f} x] - \operatorname{Log}[1 + \frac{1}{2} \sqrt{d} \sqrt{f} x]) - \\
& \quad \operatorname{PolyLog}[2, -\frac{1}{2} \sqrt{d} \sqrt{f} x] - \operatorname{PolyLog}[2, \frac{1}{2} \sqrt{d} \sqrt{f} x]) + \\
& \quad 12 b^2 d f n^2 (2 a + b n - 2 b n \operatorname{Log}[x] + 2 b \operatorname{Log}[c x^n]) \left( \frac{\operatorname{Log}[x]^3}{3} - \frac{1}{2} \operatorname{Log}[x]^2 \operatorname{Log}[1 - \frac{1}{2} \sqrt{d} \sqrt{f} x] - \right. \\
& \quad \frac{1}{2} \operatorname{Log}[x]^2 \operatorname{Log}[1 + \frac{1}{2} \sqrt{d} \sqrt{f} x] - \operatorname{Log}[x] \operatorname{PolyLog}[2, -\frac{1}{2} \sqrt{d} \sqrt{f} x] - \\
& \quad \operatorname{Log}[x] \operatorname{PolyLog}[2, \frac{1}{2} \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[3, -\frac{1}{2} \sqrt{d} \sqrt{f} x] + \operatorname{PolyLog}[3, \frac{1}{2} \sqrt{d} \sqrt{f} x]) + \\
& \quad 2 b^3 d f n^3 (\operatorname{Log}[x]^4 - 2 \operatorname{Log}[x]^3 \operatorname{Log}[1 - \frac{1}{2} \sqrt{d} \sqrt{f} x] - 2 \operatorname{Log}[x]^3 \operatorname{Log}[1 + \frac{1}{2} \sqrt{d} \sqrt{f} x] - \\
& \quad 6 \operatorname{Log}[x]^2 \operatorname{PolyLog}[2, -\frac{1}{2} \sqrt{d} \sqrt{f} x] - 6 \operatorname{Log}[x]^2 \operatorname{PolyLog}[2, \frac{1}{2} \sqrt{d} \sqrt{f} x] + \\
& \quad 12 \operatorname{Log}[x] \operatorname{PolyLog}[3, -\frac{1}{2} \sqrt{d} \sqrt{f} x] + 12 \operatorname{Log}[x] \operatorname{PolyLog}[3, \frac{1}{2} \sqrt{d} \sqrt{f} x] - \\
& \quad \left. 12 \operatorname{PolyLog}[4, -\frac{1}{2} \sqrt{d} \sqrt{f} x] - 12 \operatorname{PolyLog}[4, \frac{1}{2} \sqrt{d} \sqrt{f} x] \right)
\end{aligned}$$

### Problem 63: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Log}[\text{d} \left(\frac{1}{\text{d}} + \text{f} \sqrt{x}\right)] (a + b \text{Log}[c x^n])^3}{x^3} dx$$

Optimal (type 4, 849 leaves, 34 steps):

$$\begin{aligned}
& -\frac{175 b^3 d f n^3}{216 x^{3/2}} + \frac{45 b^3 d^2 f^2 n^3}{16 x} - \frac{255 b^3 d^3 f^3 n^3}{8 \sqrt{x}} + \frac{3}{8} b^3 d^4 f^4 n^3 \text{Log}[1 + d f \sqrt{x}] - \\
& \frac{3 b^3 n^3 \text{Log}[1 + d f \sqrt{x}]}{8 x^2} - \frac{3}{16} b^3 d^4 f^4 n^3 \text{Log}[x] + \frac{3}{16} b^3 d^4 f^4 n^3 \text{Log}[x]^2 - \\
& \frac{37 b^2 d f n^2 (a + b \text{Log}[c x^n])}{36 x^{3/2}} + \frac{21 b^2 d^2 f^2 n^2 (a + b \text{Log}[c x^n])}{8 x} - \frac{63 b^2 d^3 f^3 n^2 (a + b \text{Log}[c x^n])}{4 \sqrt{x}} + \\
& \frac{3}{4} b^2 d^4 f^4 n^2 \text{Log}[1 + d f \sqrt{x}] (a + b \text{Log}[c x^n]) - \frac{3 b^2 n^2 \text{Log}[1 + d f \sqrt{x}] (a + b \text{Log}[c x^n])}{4 x^2} - \\
& \frac{3}{8} b^2 d^4 f^4 n^2 \text{Log}[x] (a + b \text{Log}[c x^n]) - \frac{7 b d f n (a + b \text{Log}[c x^n])^2}{12 x^{3/2}} + \frac{9 b d^2 f^2 n (a + b \text{Log}[c x^n])^2}{8 x} - \\
& \frac{15 b d^3 f^3 n (a + b \text{Log}[c x^n])^2}{4 \sqrt{x}} + \frac{3}{4} b d^4 f^4 n \text{Log}[1 + d f \sqrt{x}] (a + b \text{Log}[c x^n])^2 - \\
& \frac{3 b n \text{Log}[1 + d f \sqrt{x}] (a + b \text{Log}[c x^n])^2}{4 x^2} - \frac{1}{8} d^4 f^4 (a + b \text{Log}[c x^n])^3 - \frac{d f (a + b \text{Log}[c x^n])^3}{6 x^{3/2}} + \\
& \frac{d^2 f^2 (a + b \text{Log}[c x^n])^3}{4 x} - \frac{d^3 f^3 (a + b \text{Log}[c x^n])^3}{2 \sqrt{x}} + \frac{1}{2} d^4 f^4 \text{Log}[1 + d f \sqrt{x}] (a + b \text{Log}[c x^n])^3 - \\
& \frac{\text{Log}[1 + d f \sqrt{x}] (a + b \text{Log}[c x^n])^3}{2 x^2} - \frac{d^4 f^4 (a + b \text{Log}[c x^n])^4}{16 b n} + \\
& \frac{3}{2} b^3 d^4 f^4 n^3 \text{PolyLog}[2, -d f \sqrt{x}] + 3 b^2 d^4 f^4 n^2 (a + b \text{Log}[c x^n]) \text{PolyLog}[2, -d f \sqrt{x}] + \\
& 3 b d^4 f^4 n (a + b \text{Log}[c x^n])^2 \text{PolyLog}[2, -d f \sqrt{x}] - 6 b^3 d^4 f^4 n^3 \text{PolyLog}[3, -d f \sqrt{x}] - \\
& 12 b^2 d^4 f^4 n^2 (a + b \text{Log}[c x^n]) \text{PolyLog}[3, -d f \sqrt{x}] + 24 b^3 d^4 f^4 n^3 \text{PolyLog}[4, -d f \sqrt{x}]
\end{aligned}$$

Result (type 4, 2009 leaves):

$$\begin{aligned}
& -\frac{a^3 d f}{6 x^{3/2}} - \frac{7 a^2 b d f n}{12 x^{3/2}} - \frac{37 a b^2 d f n^2}{36 x^{3/2}} - \frac{175 b^3 d f n^3}{216 x^{3/2}} + \frac{a^3 d^2 f^2}{4 x} + \frac{9 a^2 b d^2 f^2 n}{8 x} + \\
& \frac{21 a b^2 d^2 f^2 n^2}{8 x} + \frac{45 b^3 d^2 f^2 n^3}{16 x} - \frac{a^3 d^3 f^3}{2 \sqrt{x}} - \frac{15 a^2 b d^3 f^3 n}{4 \sqrt{x}} - \frac{63 a b^2 d^3 f^3 n^2}{4 \sqrt{x}} - \frac{255 b^3 d^3 f^3 n^3}{8 \sqrt{x}} + \\
& \frac{1}{2} a^3 d^4 f^4 \text{Log}[1 + d f \sqrt{x}] + \frac{3}{4} a^2 b d^4 f^4 n \text{Log}[1 + d f \sqrt{x}] + \frac{3}{4} a b^2 d^4 f^4 n^2 \text{Log}[1 + d f \sqrt{x}] + \\
& \frac{3}{8} b^3 d^4 f^4 n^3 \text{Log}[1 + d f \sqrt{x}] - \frac{a^3 \text{Log}[1 + d f \sqrt{x}]}{2 x^2} - \frac{3 a^2 b n \text{Log}[1 + d f \sqrt{x}]}{4 x^2} - \\
& \frac{3 a b^2 n^2 \text{Log}[1 + d f \sqrt{x}]}{4 x^2} - \frac{3 b^3 n^3 \text{Log}[1 + d f \sqrt{x}]}{8 x^2} - \frac{1}{4} a^3 d^4 f^4 \text{Log}[x] -
\end{aligned}$$

$$\begin{aligned}
& \frac{3}{8} a^2 b d^4 f^4 n \log[x] - \frac{3}{8} a b^2 d^4 f^4 n^2 \log[x] - \frac{3}{16} b^3 d^4 f^4 n^3 \log[x] + \frac{3}{8} a^2 b d^4 f^4 n \log[x]^2 + \\
& \frac{3}{8} a b^2 d^4 f^4 n^2 \log[x]^2 + \frac{3}{16} b^3 d^4 f^4 n^3 \log[x]^2 - \frac{1}{4} a b^2 d^4 f^4 n^2 \log[x]^3 - \frac{1}{8} b^3 d^4 f^4 n^3 \log[x]^3 + \\
& \frac{1}{2} b^3 d^4 f^4 n^3 \log[1 + \frac{1}{d f \sqrt{x}}] \log[x]^3 - \frac{1}{2} b^3 d^4 f^4 n^3 \log[1 + d f \sqrt{x}] \log[x]^3 + \\
& \frac{1}{8} b^3 d^4 f^4 n^3 \log[x]^4 - \frac{a^2 b d f \log[c x^n]}{2 x^{3/2}} - \frac{7 a b^2 d f n \log[c x^n]}{6 x^{3/2}} - \frac{37 b^3 d f n^2 \log[c x^n]}{36 x^{3/2}} + \\
& \frac{3 a^2 b d^2 f^2 \log[c x^n]}{4 x} + \frac{9 a b^2 d^2 f^2 n \log[c x^n]}{4 x} + \frac{21 b^3 d^2 f^2 n^2 \log[c x^n]}{8 x} - \\
& \frac{3 a^2 b d^3 f^3 \log[c x^n]}{2 \sqrt{x}} - \frac{15 a b^2 d^3 f^3 n \log[c x^n]}{2 \sqrt{x}} - \frac{63 b^3 d^3 f^3 n^2 \log[c x^n]}{4 \sqrt{x}} + \\
& \frac{3}{2} a^2 b d^4 f^4 \log[1 + d f \sqrt{x}] \log[c x^n] + \frac{3}{2} a b^2 d^4 f^4 n \log[1 + d f \sqrt{x}] \log[c x^n] + \\
& \frac{3}{4} b^3 d^4 f^4 n^2 \log[1 + d f \sqrt{x}] \log[c x^n] - \frac{3 a^2 b \log[1 + d f \sqrt{x}] \log[c x^n]}{2 x^2} - \\
& \frac{3 a b^2 n \log[1 + d f \sqrt{x}] \log[c x^n]}{2 x^2} - \frac{3 b^3 n^2 \log[1 + d f \sqrt{x}] \log[c x^n]}{4 x^2} - \\
& \frac{3}{4} a^2 b d^4 f^4 \log[x] \log[c x^n] - \frac{3}{4} a b^2 d^4 f^4 n \log[x] \log[c x^n] - \frac{3}{8} b^3 d^4 f^4 n^2 \log[x] \log[c x^n] + \\
& \frac{3}{4} a b^2 d^4 f^4 n \log[x]^2 \log[c x^n] + \frac{3}{8} b^3 d^4 f^4 n^2 \log[x]^2 \log[c x^n] - \frac{1}{4} b^3 d^4 f^4 n^2 \log[x]^3 \log[c x^n] - \\
& \frac{a b^2 d f \log[c x^n]^2}{2 x^{3/2}} - \frac{7 b^3 d f n \log[c x^n]^2}{12 x^{3/2}} + \frac{3 a b^2 d^2 f^2 \log[c x^n]^2}{4 x} + \frac{9 b^3 d^2 f^2 n \log[c x^n]^2}{8 x} - \\
& \frac{3 a b^2 d^3 f^3 \log[c x^n]^2}{2 \sqrt{x}} - \frac{15 b^3 d^3 f^3 n \log[c x^n]^2}{4 \sqrt{x}} + \frac{3}{2} a b^2 d^4 f^4 \log[1 + d f \sqrt{x}] \log[c x^n]^2 + \\
& \frac{3}{4} b^3 d^4 f^4 n \log[1 + d f \sqrt{x}] \log[c x^n]^2 - \frac{3 a b^2 \log[1 + d f \sqrt{x}] \log[c x^n]^2}{2 x^2} - \\
& \frac{3 b^3 n \log[1 + d f \sqrt{x}] \log[c x^n]^2}{4 x^2} - \frac{3}{4} a b^2 d^4 f^4 \log[x] \log[c x^n]^2 - \frac{3}{8} b^3 d^4 f^4 n \log[x] \log[c x^n]^2 + \\
& \frac{3}{8} b^3 d^4 f^4 n \log[x]^2 \log[c x^n]^2 - \frac{b^3 d f \log[c x^n]^3}{6 x^{3/2}} + \frac{b^3 d^2 f^2 \log[c x^n]^3}{4 x} - \\
& \frac{b^3 d^3 f^3 \log[c x^n]^3}{2 \sqrt{x}} + \frac{1}{2} b^3 d^4 f^4 \log[1 + d f \sqrt{x}] \log[c x^n]^3 - \frac{b^3 \log[1 + d f \sqrt{x}] \log[c x^n]^3}{2 x^2} - \\
& \frac{1}{4} b^3 d^4 f^4 \log[x] \log[c x^n]^3 - 3 b^3 d^4 f^4 n^3 \log[x]^2 \text{PolyLog}[2, -\frac{1}{d f \sqrt{x}}] + \\
& \frac{3}{2} b d^4 f^4 n (2 a^2 + 2 a b n + b^2 n^2 - 2 b^2 n^2 \log[x]^2 + 2 b (2 a + b n) \log[c x^n] + 2 b^2 \log[c x^n]^2) \\
& \text{PolyLog}[2, -d f \sqrt{x}] - 12 b^3 d^4 f^4 n^3 \log[x] \text{PolyLog}[3, -\frac{1}{d f \sqrt{x}}] - \\
& 12 a b^2 d^4 f^4 n^2 \text{PolyLog}[3, -d f \sqrt{x}] - 6 b^3 d^4 f^4 n^3 \text{PolyLog}[3, -d f \sqrt{x}] + \\
& 12 b^3 d^4 f^4 n^3 \log[x] \text{PolyLog}[3, -d f \sqrt{x}] -
\end{aligned}$$

$$12 b^3 d^4 f^4 n^2 \text{Log}[c x^n] \text{PolyLog}[3, -d f \sqrt{x}] - 24 b^3 d^4 f^4 n^3 \text{PolyLog}[4, -\frac{1}{d f \sqrt{x}}]$$

**Problem 64: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \text{Log}[c x^n])^4 \text{Log}[d (\frac{1}{d} + f x^m)]}{x} dx$$

Optimal (type 4, 137 leaves, 5 steps) :

$$\begin{aligned} & -\frac{(a + b \text{Log}[c x^n])^4 \text{PolyLog}[2, -d f x^m]}{m} + \\ & \frac{4 b n (a + b \text{Log}[c x^n])^3 \text{PolyLog}[3, -d f x^m]}{m^2} - \frac{12 b^2 n^2 (a + b \text{Log}[c x^n])^2 \text{PolyLog}[4, -d f x^m]}{m^3} + \\ & \frac{24 b^3 n^3 (a + b \text{Log}[c x^n]) \text{PolyLog}[5, -d f x^m]}{m^4} - \frac{24 b^4 n^4 \text{PolyLog}[6, -d f x^m]}{m^5} \end{aligned}$$

Result (type 4, 1700 leaves) :

$$\begin{aligned} & -\frac{2}{3} a^3 b m n \text{Log}[x]^3 + \frac{3}{2} a^2 b^2 m n^2 \text{Log}[x]^4 - \frac{6}{5} a b^3 m n^3 \text{Log}[x]^5 + \frac{1}{3} b^4 m n^4 \text{Log}[x]^6 - \\ & 2 a^2 b^2 m n \text{Log}[x]^3 \text{Log}[c x^n] + 3 a b^3 m n^2 \text{Log}[x]^4 \text{Log}[c x^n] - \frac{6}{5} b^4 m n^3 \text{Log}[x]^5 \text{Log}[c x^n] - \\ & 2 a b^3 m n \text{Log}[x]^3 \text{Log}[c x^n]^2 + \frac{3}{2} b^4 m n^2 \text{Log}[x]^4 \text{Log}[c x^n]^2 - \frac{2}{3} b^4 m n \text{Log}[x]^3 \text{Log}[c x^n]^3 - \\ & 2 a^3 b n \text{Log}[x]^2 \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] + 4 a^2 b^2 n^2 \text{Log}[x]^3 \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] - 3 a b^3 n^3 \text{Log}[x]^4 \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] + \\ & \frac{4}{5} b^4 n^4 \text{Log}[x]^5 \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] - 6 a^2 b^2 n \text{Log}[x]^2 \text{Log}[c x^n] \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] + \\ & 8 a b^3 n^2 \text{Log}[x]^3 \text{Log}[c x^n] \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] - 3 b^4 n^3 \text{Log}[x]^4 \text{Log}[c x^n] \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] - \\ & 6 a b^3 n \text{Log}[x]^2 \text{Log}[c x^n]^2 \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] + 4 b^4 n^2 \text{Log}[x]^3 \text{Log}[c x^n]^2 \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] - \\ & 2 b^4 n \text{Log}[x]^2 \text{Log}[c x^n]^3 \text{Log}\left[1 + \frac{x^{-m}}{d f}\right] + 2 a^3 b n \text{Log}[x]^2 \text{Log}\left[1 + d f x^m\right] - \\ & 4 a^2 b^2 n^2 \text{Log}[x]^3 \text{Log}\left[1 + d f x^m\right] + 3 a b^3 n^3 \text{Log}[x]^4 \text{Log}\left[1 + d f x^m\right] - \\ & \frac{4}{5} b^4 n^4 \text{Log}[x]^5 \text{Log}\left[1 + d f x^m\right] + \frac{a^4 \text{Log}[-d f x^m] \text{Log}[1 + d f x^m]}{m} - \\ & \frac{4 a^3 b n \text{Log}[x] \text{Log}[-d f x^m] \text{Log}[1 + d f x^m]}{m} + \frac{6 a^2 b^2 n^2 \text{Log}[x]^2 \text{Log}[-d f x^m] \text{Log}[1 + d f x^m]}{m} - \\ & 4 a b^3 n^3 \text{Log}[x]^3 \text{Log}[-d f x^m] \text{Log}[1 + d f x^m] + \frac{b^4 n^4 \text{Log}[x]^4 \text{Log}[-d f x^m] \text{Log}[1 + d f x^m]}{m} + \\ & 6 a^2 b^2 n \text{Log}[x]^2 \text{Log}[c x^n] \text{Log}\left[1 + d f x^m\right] - 8 a b^3 n^2 \text{Log}[x]^3 \text{Log}[c x^n] \text{Log}\left[1 + d f x^m\right] + \\ & 3 b^4 n^3 \text{Log}[x]^4 \text{Log}[c x^n] \text{Log}\left[1 + d f x^m\right] + \frac{4 a^3 b \text{Log}[-d f x^m] \text{Log}[c x^n] \text{Log}\left[1 + d f x^m\right]}{m} - \\ & \frac{12 a^2 b^2 n \text{Log}[x] \text{Log}[-d f x^m] \text{Log}[c x^n] \text{Log}\left[1 + d f x^m\right]}{m} + \end{aligned}$$

$$\begin{aligned}
& \frac{12 a b^3 n^2 \log[x]^2 \log[-d f x^m] \log[c x^n] \log[1 + d f x^m]}{m} - \\
& \frac{4 b^4 n^3 \log[x]^3 \log[-d f x^m] \log[c x^n] \log[1 + d f x^m]}{m} + 6 a b^3 n \log[x]^2 \log[c x^n]^2 \log[1 + d f x^m] - \\
& \frac{4 b^4 n^2 \log[x]^3 \log[c x^n]^2 \log[1 + d f x^m]}{m} + \frac{6 a^2 b^2 \log[-d f x^m] \log[c x^n]^2 \log[1 + d f x^m]}{m} - \\
& \frac{12 a b^3 n \log[x] \log[-d f x^m] \log[c x^n]^2 \log[1 + d f x^m]}{m} + \\
& \frac{6 b^4 n^2 \log[x]^2 \log[-d f x^m] \log[c x^n]^2 \log[1 + d f x^m]}{m} + \\
& 2 b^4 n \log[x]^2 \log[c x^n]^3 \log[1 + d f x^m] + \frac{4 a b^3 \log[-d f x^m] \log[c x^n]^3 \log[1 + d f x^m]}{m} - \\
& \frac{4 b^4 n \log[x] \log[-d f x^m] \log[c x^n]^3 \log[1 + d f x^m]}{m} + \frac{b^4 \log[-d f x^m] \log[c x^n]^4 \log[1 + d f x^m]}{m} + \\
& \frac{1}{m} b n \log[x] \left( -b^3 n^3 \log[x]^3 + 4 b^2 n^2 \log[x]^2 (a + b \log[c x^n]) \right) - \\
& 6 b n \log[x] (a + b \log[c x^n])^2 + 4 (a + b \log[c x^n])^3 \text{PolyLog}[2, -\frac{x^{-m}}{d f}] + \\
& \frac{(a - b n \log[x] + b \log[c x^n])^4 \text{PolyLog}[2, 1 + d f x^m]}{m} + \frac{4 a^3 b n \text{PolyLog}[3, -\frac{x^{-m}}{d f}]}{m^2} + \\
& \frac{12 a^2 b^2 n \log[c x^n] \text{PolyLog}[3, -\frac{x^{-m}}{d f}]}{m^2} + \frac{12 a b^3 n \log[c x^n]^2 \text{PolyLog}[3, -\frac{x^{-m}}{d f}]}{m^2} + \\
& \frac{4 b^4 n \log[c x^n]^3 \text{PolyLog}[3, -\frac{x^{-m}}{d f}]}{m^2} + \frac{12 a^2 b^2 n^2 \text{PolyLog}[4, -\frac{x^{-m}}{d f}]}{m^3} + \\
& \frac{24 a b^3 n^2 \log[c x^n] \text{PolyLog}[4, -\frac{x^{-m}}{d f}]}{m^3} + \frac{12 b^4 n^2 \log[c x^n]^2 \text{PolyLog}[4, -\frac{x^{-m}}{d f}]}{m^3} + \\
& \frac{24 a b^3 n^3 \text{PolyLog}[5, -\frac{x^{-m}}{d f}]}{m^4} + \frac{24 b^4 n^3 \log[c x^n] \text{PolyLog}[5, -\frac{x^{-m}}{d f}]}{m^4} + \frac{24 b^4 n^4 \text{PolyLog}[6, -\frac{x^{-m}}{d f}]}{m^5}
\end{aligned}$$

**Problem 65: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \log[c x^n])^3 \log[d (\frac{1}{d} + f x^m)]}{x} dx$$

Optimal (type 4, 105 leaves, 4 steps) :

$$\begin{aligned}
& - \frac{(a + b \log[c x^n])^3 \text{PolyLog}[2, -d f x^m]}{m} + \frac{3 b n (a + b \log[c x^n])^2 \text{PolyLog}[3, -d f x^m]}{m^2} - \\
& \frac{6 b^2 n^2 (a + b \log[c x^n]) \text{PolyLog}[4, -d f x^m]}{m^3} + \frac{6 b^3 n^3 \text{PolyLog}[5, -d f x^m]}{m^4}
\end{aligned}$$

Result (type 4, 1035 leaves) :

$$\begin{aligned}
& -\frac{1}{2} a^2 b m n \operatorname{Log}[x]^3 + \frac{3}{4} a b^2 m n^2 \operatorname{Log}[x]^4 - \frac{3}{10} b^3 m n^3 \operatorname{Log}[x]^5 - \\
& a b^2 m n \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] + \frac{3}{4} b^3 m n^2 \operatorname{Log}[x]^4 \operatorname{Log}[c x^n] - \frac{1}{2} b^3 m n \operatorname{Log}[x]^3 \operatorname{Log}[c x^n]^2 - \\
& \frac{3}{2} a^2 b n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] + 2 a b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] - \frac{3}{4} b^3 n^3 \operatorname{Log}[x]^4 \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] - \\
& 3 a b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] + 2 b^3 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] - \\
& \frac{3}{2} b^3 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] + \frac{3}{2} a^2 b n \operatorname{Log}[x]^2 \operatorname{Log}[1 + d f x^m] - \\
& 2 a b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[1 + d f x^m] + \frac{3}{4} b^3 n^3 \operatorname{Log}[x]^4 \operatorname{Log}[1 + d f x^m] + \frac{a^3 \operatorname{Log}[-d f x^m] \operatorname{Log}[1 + d f x^m]}{m} - \\
& \frac{3 a^2 b n \operatorname{Log}[x] \operatorname{Log}[-d f x^m] \operatorname{Log}[1 + d f x^m]}{m} + \frac{3 a b^2 n^2 \operatorname{Log}[x]^2 \operatorname{Log}[-d f x^m] \operatorname{Log}[1 + d f x^m]}{m} - \\
& \frac{b^3 n^3 \operatorname{Log}[x]^3 \operatorname{Log}[-d f x^m] \operatorname{Log}[1 + d f x^m]}{m} + 3 a b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}[1 + d f x^m] - \\
& 2 b^3 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}[1 + d f x^m] + \frac{3 a^2 b \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n] \operatorname{Log}[1 + d f x^m]}{m} - \\
& \frac{6 a b^2 n \operatorname{Log}[x] \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n] \operatorname{Log}[1 + d f x^m]}{m} + \\
& \frac{3 b^3 n^2 \operatorname{Log}[x]^2 \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n] \operatorname{Log}[1 + d f x^m]}{m} + \\
& \frac{3 b^3 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + d f x^m]}{m} + \frac{3 a b^2 \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + d f x^m]}{m} - \\
& \frac{3 b^3 n \operatorname{Log}[x] \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + d f x^m]}{m} + \frac{b^3 \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n]^3 \operatorname{Log}[1 + d f x^m]}{m} + \frac{1}{m} \\
& b n \operatorname{Log}[x] \left( b^2 n^2 \operatorname{Log}[x]^2 - 3 b n \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n]) + 3 (a + b \operatorname{Log}[c x^n])^2 \right) \operatorname{PolyLog}[2, -\frac{x^{-m}}{d f}] + \\
& \frac{(a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}[2, 1 + d f x^m]}{m} + \frac{3 a^2 b n \operatorname{PolyLog}[3, -\frac{x^{-m}}{d f}]}{m^2} + \\
& \frac{6 a b^2 n \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, -\frac{x^{-m}}{d f}]}{m^2} + \frac{3 b^3 n \operatorname{Log}[c x^n]^2 \operatorname{PolyLog}[3, -\frac{x^{-m}}{d f}]}{m^2} + \\
& \frac{6 a b^2 n^2 \operatorname{PolyLog}[4, -\frac{x^{-m}}{d f}]}{m^3} + \frac{6 b^3 n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[4, -\frac{x^{-m}}{d f}]}{m^3} + \frac{6 b^3 n^3 \operatorname{PolyLog}[5, -\frac{x^{-m}}{d f}]}{m^4}
\end{aligned}$$

**Problem 66: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (\frac{1}{d} + f x^m)]}{x} dx$$

Optimal (type 4, 73 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{(a+b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -d f x^m]}{m} + \\
 & \frac{2 b n (a+b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -d f x^m]}{m^2} - \frac{2 b^2 n^2 \operatorname{PolyLog}[4, -d f x^m]}{m^3}
 \end{aligned}$$

Result (type 4, 526 leaves) :

$$\begin{aligned}
 & -\frac{1}{3} a b m n \operatorname{Log}[x]^3 + \frac{1}{4} b^2 m n^2 \operatorname{Log}[x]^4 - \frac{1}{3} b^2 m n \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] - \\
 & a b n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] + \frac{2}{3} b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] - b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] + \\
 & a b n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + d f x^m\right] - \frac{2}{3} b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + d f x^m\right] + \frac{a^2 \operatorname{Log}[-d f x^m] \operatorname{Log}[1 + d f x^m]}{m} - \\
 & \frac{2 a b n \operatorname{Log}[x] \operatorname{Log}[-d f x^m] \operatorname{Log}[1 + d f x^m]}{m} + \frac{b^2 n^2 \operatorname{Log}[x]^2 \operatorname{Log}[-d f x^m] \operatorname{Log}[1 + d f x^m]}{m} + \\
 & b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + d f x^m\right] + \frac{2 a b \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n] \operatorname{Log}[1 + d f x^m]}{m} - \\
 & \frac{2 b^2 n \operatorname{Log}[x] \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n] \operatorname{Log}[1 + d f x^m]}{m} + \frac{b^2 \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n]^2 \operatorname{Log}[1 + d f x^m]}{m} + \\
 & \frac{b n \operatorname{Log}[x] (-b n \operatorname{Log}[x] + 2 (a + b \operatorname{Log}[c x^n])) \operatorname{PolyLog}\left[2, -\frac{x^{-m}}{d f}\right]}{m} + \\
 & \frac{(a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, 1 + d f x^m]}{m} + \frac{2 a b n \operatorname{PolyLog}\left[3, -\frac{x^{-m}}{d f}\right]}{m^2} + \\
 & \frac{2 b^2 n \operatorname{Log}[c x^n] \operatorname{PolyLog}\left[3, -\frac{x^{-m}}{d f}\right]}{m^2} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[4, -\frac{x^{-m}}{d f}\right]}{m^3}
 \end{aligned}$$

Problem 67: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[d\left(\frac{1}{d} + f x^m\right)\right]}{x} dx$$

Optimal (type 4, 40 leaves, 2 steps) :

$$-\frac{(a+b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -d f x^m]}{m} + \frac{b n \operatorname{PolyLog}[3, -d f x^m]}{m^2}$$

Result (type 4, 207 leaves) :

$$\begin{aligned}
 & \frac{1}{6 m^2} \left( -b m^3 n \operatorname{Log}[x]^3 - 3 b m^2 n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{x^{-m}}{d f}\right] + 3 b m^2 n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + d f x^m\right] + \right. \\
 & 6 a m \operatorname{Log}[-d f x^m] \operatorname{Log}\left[1 + d f x^m\right] - 6 b m n \operatorname{Log}[x] \operatorname{Log}[-d f x^m] \operatorname{Log}\left[1 + d f x^m\right] + \\
 & 6 b m \operatorname{Log}[-d f x^m] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + d f x^m\right] + 6 b m n \operatorname{Log}[x] \operatorname{PolyLog}\left[2, -\frac{x^{-m}}{d f}\right] + \\
 & \left. 6 m (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, 1 + d f x^m\right] + 6 b n \operatorname{PolyLog}\left[3, -\frac{x^{-m}}{d f}\right] \right)
 \end{aligned}$$

### Problem 81: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x} dx$$

Optimal (type 4, 131 leaves, 5 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{3 b n} - \\ & \frac{m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{3 b n} - m (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right] + \\ & 2 b m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right] - 2 b^2 m n^2 \operatorname{PolyLog}\left[4, -\frac{f x}{e}\right] \end{aligned}$$

Result (type 4, 329 leaves):

$$\begin{aligned} & a^2 \operatorname{Log}[x] \operatorname{Log}[d (e + f x)^m] - a b n \operatorname{Log}[x]^2 \operatorname{Log}[d (e + f x)^m] + \\ & \frac{1}{3} b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[d (e + f x)^m] + 2 a b \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] - \\ & b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] + b^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x)^m] - \\ & a^2 m \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{f x}{e}\right] + a b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right] - \frac{1}{3} b^2 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{f x}{e}\right] - \\ & 2 a b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{f x}{e}\right] + b^2 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{f x}{e}\right] - \\ & b^2 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right] - m (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right] + \\ & 2 b m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right] - 2 b^2 m n^2 \operatorname{PolyLog}\left[4, -\frac{f x}{e}\right] \end{aligned}$$

### Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x^2} dx$$

Optimal (type 4, 248 leaves, 10 steps):

$$\begin{aligned} & \frac{2 b^2 f m n^2 \operatorname{Log}[x]}{e} - \frac{2 b f m n \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])}{e} - \frac{f m \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^2}{e} - \\ & \frac{2 b^2 f m n^2 \operatorname{Log}[e + f x]}{e} - \frac{2 b^2 n^2 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{x} - \\ & \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x} + \frac{2 b^2 f m n^2 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e} + \\ & \frac{2 b f m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e} + \frac{2 b^2 f m n^2 \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{e} \end{aligned}$$

Result (type 4, 600 leaves):

$$\begin{aligned}
& -\frac{1}{3ex} \left( -3a^2 f m x \operatorname{Log}[x] - 6abfmn x \operatorname{Log}[x] - 6b^2 fm n^2 x \operatorname{Log}[x] + 3abfmn x \operatorname{Log}[x]^2 + \right. \\
& \quad 3b^2 fm n^2 x \operatorname{Log}[x]^2 - b^2 fm n^2 x \operatorname{Log}[x]^3 - 6abfm x \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
& \quad 6b^2 fm n x \operatorname{Log}[c x^n] + 3b^2 fm n x \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] - 3b^2 fm x \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + \\
& \quad 3a^2 fm x \operatorname{Log}[e + fx] + 6abfmn x \operatorname{Log}[e + fx] + 6b^2 fm n^2 x \operatorname{Log}[e + fx] - \\
& \quad 6abfmn x \operatorname{Log}[x] \operatorname{Log}[e + fx] - 6b^2 fm n^2 x \operatorname{Log}[e + fx] + \\
& \quad 3b^2 fm n^2 x \operatorname{Log}[x]^2 \operatorname{Log}[e + fx] + 6abfm x \operatorname{Log}[c x^n] \operatorname{Log}[e + fx] + \\
& \quad 6b^2 fm n x \operatorname{Log}[c x^n] \operatorname{Log}[e + fx] - 6b^2 fm n x \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[e + fx] + \\
& \quad 3b^2 fm x \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + fx] + 3a^2 e \operatorname{Log}[d(e + fx)^m] + 6aben \operatorname{Log}[d(e + fx)^m] + \\
& \quad 6b^2 en \operatorname{Log}[d(e + fx)^m] + 6abe \operatorname{Log}[c x^n] \operatorname{Log}[d(e + fx)^m] + \\
& \quad 6b^2 en \operatorname{Log}[c x^n] \operatorname{Log}[d(e + fx)^m] + 3b^2 e \operatorname{Log}[c x^n]^2 \operatorname{Log}[d(e + fx)^m] + \\
& \quad 6abfmn x \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{fx}{e}\right] + 6b^2 fm n^2 x \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{fx}{e}\right] - \\
& \quad \left. 3b^2 fm n^2 x \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{fx}{e}\right] + 6b^2 fm n x \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{fx}{e}\right] + \right. \\
& \quad \left. 6b fm n x (a + bn + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{fx}{e}] - 6b^2 fm n^2 x \operatorname{PolyLog}[3, -\frac{fx}{e}] \right)
\end{aligned}$$

**Problem 83: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d(e + fx)^m]}{x^3} dx$$

Optimal (type 4, 344 leaves, 14 steps):

$$\begin{aligned}
& -\frac{7b^2 fm n^2}{4ex} - \frac{b^2 f^2 m n^2 \operatorname{Log}[x]}{4e^2} - \frac{3bfmn(a + b \operatorname{Log}[c x^n])}{2ex} + \frac{bf^2 mn \operatorname{Log}\left[1 + \frac{e}{fx}\right] (a + b \operatorname{Log}[c x^n])}{2e^2} - \\
& \frac{fm(a + b \operatorname{Log}[c x^n])^2}{2ex} + \frac{f^2 m \operatorname{Log}\left[1 + \frac{e}{fx}\right] (a + b \operatorname{Log}[c x^n])^2}{2e^2} + \frac{b^2 f^2 m n^2 \operatorname{Log}[e + fx]}{4e^2} - \\
& \frac{b^2 n^2 \operatorname{Log}[d(e + fx)^m]}{4x^2} - \frac{bn(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d(e + fx)^m]}{2x^2} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d(e + fx)^m]}{2x^2} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}[2, -\frac{e}{fx}]}{2e^2} - \\
& \frac{bf^2 mn(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{e}{fx}]}{e^2} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}[3, -\frac{e}{fx}]}{e^2}
\end{aligned}$$

Result (type 4, 796 leaves):

$$\begin{aligned}
& -\frac{1}{12 e^2 x^2} \left( 6 a^2 e f m x + 18 a b e f m n x + 21 b^2 e f m n^2 x + 6 a^2 f^2 m x^2 \log[x] + \right. \\
& \quad 6 a b f^2 m n x^2 \log[x] + 3 b^2 f^2 m n^2 x^2 \log[x] - 6 a b f^2 m n x^2 \log[x]^2 - \\
& \quad 3 b^2 f^2 m n^2 x^2 \log[x]^2 + 2 b^2 f^2 m n^2 x^2 \log[x]^3 + 12 a b e f m x \log[c x^n] + \\
& \quad 18 b^2 e f m n x \log[c x^n] + 12 a b f^2 m n x^2 \log[x] \log[c x^n] + 6 b^2 f^2 m n x^2 \log[x] \log[c x^n] - \\
& \quad 6 b^2 f^2 m n x^2 \log[x]^2 \log[c x^n] + 6 b^2 e f m x \log[c x^n]^2 + 6 b^2 f^2 m n x^2 \log[x] \log[c x^n]^2 - \\
& \quad 6 a^2 f^2 m n x^2 \log[e + f x] - 6 a b f^2 m n x^2 \log[e + f x] - 3 b^2 f^2 m n^2 x^2 \log[e + f x] + \\
& \quad 12 a b f^2 m n x^2 \log[x] \log[e + f x] + 6 b^2 f^2 m n^2 x^2 \log[x] \log[e + f x] - \\
& \quad 6 b^2 f^2 m n^2 x^2 \log[e + f x] - 12 a b f^2 m n x^2 \log[c x^n] \log[e + f x] - \\
& \quad 6 b^2 f^2 m n x^2 \log[c x^n] \log[e + f x] + 12 b^2 f^2 m n x^2 \log[x] \log[c x^n] \log[e + f x] - \\
& \quad 6 b^2 f^2 m n x^2 \log[c x^n]^2 \log[e + f x] + 6 a^2 e^2 \log[d(e + f x)^m] + 6 a b e^2 n \log[d(e + f x)^m] + \\
& \quad 3 b^2 e^2 n^2 \log[d(e + f x)^m] + 12 a b e^2 \log[c x^n] \log[d(e + f x)^m] + \\
& \quad 6 b^2 e^2 n \log[c x^n] \log[d(e + f x)^m] + 6 b^2 e^2 \log[c x^n]^2 \log[d(e + f x)^m] - \\
& \quad 12 a b f^2 m n x^2 \log[x] \log\left[1 + \frac{f x}{e}\right] - 6 b^2 f^2 m n^2 x^2 \log[x] \log\left[1 + \frac{f x}{e}\right] + \\
& \quad 6 b^2 f^2 m n^2 x^2 \log[x]^2 \log\left[1 + \frac{f x}{e}\right] - 12 b^2 f^2 m n x^2 \log[x] \log[c x^n] \log\left[1 + \frac{f x}{e}\right] - \\
& \quad 6 b f^2 m n x^2 (2 a + b n + 2 b \log[c x^n]) \operatorname{PolyLog}[2, -\frac{f x}{e}] + 12 b^2 f^2 m n^2 x^2 \operatorname{PolyLog}[3, -\frac{f x}{e}] \Big)
\end{aligned}$$

**Problem 84: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \log[c x^n])^2 \log[d(e + f x)^m]}{x^4} dx$$

Optimal (type 4, 420 leaves, 19 steps):

$$\begin{aligned}
& -\frac{19 b^2 f m n^2}{108 e x^2} + \frac{26 b^2 f^2 m n^2}{27 e^2 x} + \frac{2 b^2 f^3 m n^2 \log[x]}{27 e^3} - \frac{5 b f m n (a + b \log[c x^n])}{18 e x^2} + \\
& \frac{8 b f^2 m n (a + b \log[c x^n])}{9 e^2 x} - \frac{2 b f^3 m n \log\left[1 + \frac{e}{f x}\right] (a + b \log[c x^n])}{9 e^3} - \\
& \frac{f m (a + b \log[c x^n])^2}{6 e x^2} + \frac{f^2 m (a + b \log[c x^n])^2}{3 e^2 x} - \frac{f^3 m \log\left[1 + \frac{e}{f x}\right] (a + b \log[c x^n])^2}{3 e^3} - \\
& \frac{2 b^2 f^3 m n^2 \log[e + f x]}{27 e^3} - \frac{2 b^2 n^2 \log[d(e + f x)^m]}{27 x^3} - \frac{2 b n (a + b \log[c x^n]) \log[d(e + f x)^m]}{9 x^3} - \\
& \frac{(a + b \log[c x^n])^2 \log[d(e + f x)^m]}{3 x^3} + \frac{2 b^2 f^3 m n^2 \operatorname{PolyLog}[2, -\frac{e}{f x}]}{9 e^3} + \\
& \frac{2 b f^3 m n (a + b \log[c x^n]) \operatorname{PolyLog}[2, -\frac{e}{f x}]}{3 e^3} + \frac{2 b^2 f^3 m n^2 \operatorname{PolyLog}[3, -\frac{e}{f x}]}{3 e^3}
\end{aligned}$$

Result (type 4, 909 leaves):

$$\begin{aligned}
& - \frac{1}{108 e^3 x^3} \left( 18 a^2 e^2 f m x + 30 a b e^2 f m n x + 19 b^2 e^2 f m n^2 x - 36 a^2 e f^2 m x^2 - \right. \\
& \quad 96 a b e f^2 m n x^2 - 104 b^2 e f^2 m n^2 x^2 - 36 a^2 f^3 m x^3 \operatorname{Log}[x] - 24 a b f^3 m n x^3 \operatorname{Log}[x] - \\
& \quad 8 b^2 f^3 m n^2 x^3 \operatorname{Log}[x] + 36 a b f^3 m n x^3 \operatorname{Log}[x]^2 + 12 b^2 f^3 m n^2 x^3 \operatorname{Log}[x]^2 - \\
& \quad 12 b^2 f^3 m n^2 x^3 \operatorname{Log}[x]^3 + 36 a b e^2 f m x \operatorname{Log}[c x^n] + 30 b^2 e^2 f m n x \operatorname{Log}[c x^n] - \\
& \quad 72 a b e f^2 m x^2 \operatorname{Log}[c x^n] - 96 b^2 e f^2 m n x^2 \operatorname{Log}[c x^n] - 72 a b f^3 m x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
& \quad 24 b^2 f^3 m n x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] + 36 b^2 f^3 m n x^3 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + \\
& \quad 18 b^2 e^2 f m x \operatorname{Log}[c x^n]^2 - 36 b^2 e f^2 m x^2 \operatorname{Log}[c x^n]^2 - 36 b^2 f^3 m x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + \\
& \quad 36 a^2 f^3 m x^3 \operatorname{Log}[e + f x] + 24 a b f^3 m n x^3 \operatorname{Log}[e + f x] + 8 b^2 f^3 m n^2 x^3 \operatorname{Log}[e + f x] - \\
& \quad 72 a b f^3 m n x^3 \operatorname{Log}[x] \operatorname{Log}[e + f x] - 24 b^2 f^3 m n^2 x^3 \operatorname{Log}[x] \operatorname{Log}[e + f x] + \\
& \quad 36 b^2 f^3 m n^2 x^3 \operatorname{Log}[x]^2 \operatorname{Log}[e + f x] + 72 a b f^3 m x^3 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x] + \\
& \quad 24 b^2 f^3 m n x^3 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x] - 72 b^2 f^3 m n x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[e + f x] + \\
& \quad 36 b^2 f^3 m x^3 \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x] + 36 a^2 e^3 \operatorname{Log}[d (e + f x)^m] + 24 a b e^3 n \operatorname{Log}[d (e + f x)^m] + \\
& \quad 8 b^2 e^3 n^2 \operatorname{Log}[d (e + f x)^m] + 72 a b e^3 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] + \\
& \quad 24 b^2 e^3 n \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] + 36 b^2 e^3 \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x)^m] + \\
& \quad 72 a b f^3 m n x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{f x}{e}\right] + 24 b^2 f^3 m n^2 x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{f x}{e}\right] - \\
& \quad 36 b^2 f^3 m n^2 x^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right] + 72 b^2 f^3 m n x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{f x}{e}\right] + \\
& \quad \left. 24 b f^3 m n x^3 (3 a + b n + 3 b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{f x}{e}] - 72 b^2 f^3 m n^2 x^3 \operatorname{PolyLog}[3, -\frac{f x}{e}] \right)
\end{aligned}$$

**Problem 85: Result more than twice size of optimal antiderivative.**

$$\int x (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m] dx$$

Optimal (type 4, 603 leaves, 34 steps):

$$\begin{aligned}
& \frac{21 a b^2 e m n^2 x}{4 f} - \frac{45 b^3 e m n^3 x}{8 f} + \frac{3}{4} b^3 m n^3 x^2 + \frac{21 b^3 e m n^2 x \log[c x^n]}{4 f} - \frac{9}{8} b^2 m n^2 x^2 (a + b \log[c x^n]) - \\
& \frac{9 b e m n x (a + b \log[c x^n])^2}{4 f} + \frac{3}{4} b m n x^2 (a + b \log[c x^n])^2 + \frac{e m x (a + b \log[c x^n])^3}{2 f} - \\
& \frac{1}{4} m x^2 (a + b \log[c x^n])^3 + \frac{3 b^3 e^2 m n^3 \log[e + f x]}{8 f^2} - \frac{3}{8} b^3 n^3 x^2 \log[d (e + f x)^m] + \\
& \frac{3}{4} b^2 n^2 x^2 (a + b \log[c x^n]) \log[d (e + f x)^m] - \frac{3}{4} b n x^2 (a + b \log[c x^n])^2 \log[d (e + f x)^m] + \\
& \frac{1}{2} x^2 (a + b \log[c x^n])^3 \log[d (e + f x)^m] - \frac{3 b^2 e^2 m n^2 (a + b \log[c x^n]) \log[1 + \frac{f x}{e}]}{4 f^2} + \\
& \frac{3 b e^2 m n (a + b \log[c x^n])^2 \log[1 + \frac{f x}{e}]}{4 f^2} - \frac{e^2 m (a + b \log[c x^n])^3 \log[1 + \frac{f x}{e}]}{2 f^2} - \\
& \frac{3 b^3 e^2 m n^3 \text{PolyLog}[2, -\frac{f x}{e}]}{4 f^2} + \frac{3 b^2 e^2 m n^2 (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{f x}{e}]}{2 f^2} - \\
& \frac{3 b e^2 m n (a + b \log[c x^n])^2 \text{PolyLog}[2, -\frac{f x}{e}]}{2 f^2} - \frac{3 b^3 e^2 m n^3 \text{PolyLog}[3, -\frac{f x}{e}]}{2 f^2} + \\
& \frac{3 b^2 e^2 m n^2 (a + b \log[c x^n]) \text{PolyLog}[3, -\frac{f x}{e}]}{f^2} - \frac{3 b^3 e^2 m n^3 \text{PolyLog}[4, -\frac{f x}{e}]}{f^2}
\end{aligned}$$

Result (type 4, 1431 leaves):

$$\begin{aligned}
& \frac{1}{8 f^2} \left( 4 a^3 e f m x - 18 a^2 b e f m n x + 42 a b^2 e f m n^2 x - 45 b^3 e f m n^3 x - \right. \\
& \quad 2 a^3 f^2 m x^2 + 6 a^2 b f^2 m n x^2 - 9 a b^2 f^2 m n^2 x^2 + 6 b^3 f^2 m n^3 x^2 + 12 a^2 b e f m x \log[c x^n] - \\
& \quad 36 a b^2 e f m n x \log[c x^n] + 42 b^3 e f m n^2 x \log[c x^n] - 6 a^2 b f^2 m x^2 \log[c x^n] + \\
& \quad 12 a b^2 f^2 m n x^2 \log[c x^n] - 9 b^3 f^2 m n^2 x^2 \log[c x^n] + 12 a b^2 e f m x \log[c x^n]^2 - \\
& \quad 18 b^3 e f m n x \log[c x^n]^2 - 6 a b^2 f^2 m x^2 \log[c x^n]^2 + 6 b^3 f^2 m n x^2 \log[c x^n]^2 + \\
& \quad 4 b^3 e f m x \log[c x^n]^3 - 2 b^3 f^2 m x^2 \log[c x^n]^3 - 4 a^3 e^2 m \log[e + f x] + 6 a^2 b e^2 m n \log[e + f x] - \\
& \quad 6 a b^2 e^2 m n^2 \log[e + f x] + 3 b^3 e^2 m n^3 \log[e + f x] + 12 a^2 b e^2 m n \log[x] \log[e + f x] - \\
& \quad 12 a b^2 e^2 m n^2 \log[x] \log[e + f x] + 6 b^3 e^2 m n^3 \log[x] \log[e + f x] - \\
& \quad 12 a b^2 e^2 m n^2 \log[x]^2 \log[e + f x] + 6 b^3 e^2 m n^3 \log[x]^2 \log[e + f x] + \\
& \quad 4 b^3 e^2 m n^3 \log[x]^3 \log[e + f x] - 12 a^2 b e^2 m \log[c x^n] \log[e + f x] + \\
& \quad 12 a b^2 e^2 m n \log[c x^n] \log[e + f x] - 6 b^3 e^2 m n^2 \log[c x^n] \log[e + f x] + \\
& \quad 24 a b^2 e^2 m n \log[x] \log[c x^n] \log[e + f x] - 12 b^3 e^2 m n^2 \log[x] \log[c x^n] \log[e + f x] - \\
& \quad 12 b^3 e^2 m n^2 \log[x]^2 \log[c x^n] \log[e + f x] - 12 a b^2 e^2 m \log[c x^n]^2 \log[e + f x] + \\
& \quad 6 b^3 e^2 m n \log[c x^n]^2 \log[e + f x] + 12 b^3 e^2 m n \log[x] \log[c x^n]^2 \log[e + f x] - \\
& \quad 4 b^3 e^2 m \log[c x^n]^3 \log[e + f x] + 4 a^3 f^2 x^2 \log[d (e + f x)^m] - 6 a^2 b f^2 n x^2 \log[d (e + f x)^m] + \\
& \quad 6 a b^2 f^2 n^2 x^2 \log[d (e + f x)^m] - 3 b^3 f^2 n^3 x^2 \log[d (e + f x)^m] + \\
& \quad 12 a^2 b f^2 x^2 \log[c x^n] \log[d (e + f x)^m] - 12 a b^2 f^2 n x^2 \log[c x^n] \log[d (e + f x)^m] - \\
& \quad 6 b^3 f^2 n^2 x^2 \log[c x^n] \log[d (e + f x)^m] + 12 a b^2 f^2 x^2 \log[c x^n]^2 \log[d (e + f x)^m] - \\
& \quad 6 b^3 f^2 n x^2 \log[c x^n]^2 \log[d (e + f x)^m] + 4 b^3 f^2 x^2 \log[c x^n]^3 \log[d (e + f x)^m] - \\
& \quad 12 a^2 b e^2 m n \log[x] \log\left[1 + \frac{f x}{e}\right] + 12 a b^2 e^2 m n^2 \log[x] \log\left[1 + \frac{f x}{e}\right] - \\
& \quad 6 b^3 e^2 m n^3 \log[x] \log\left[1 + \frac{f x}{e}\right] + 12 a b^2 e^2 m n^2 \log[x]^2 \log\left[1 + \frac{f x}{e}\right] - \\
& \quad 6 b^3 e^2 m n^3 \log[x]^2 \log\left[1 + \frac{f x}{e}\right] - 4 b^3 e^2 m n^3 \log[x]^3 \log\left[1 + \frac{f x}{e}\right] - \\
& \quad 24 a b^2 e^2 m n \log[x] \log[c x^n] \log\left[1 + \frac{f x}{e}\right] + 12 b^3 e^2 m n^2 \log[x] \log[c x^n] \log\left[1 + \frac{f x}{e}\right] + \\
& \quad 12 b^3 e^2 m n^2 \log[x]^2 \log[c x^n] \log\left[1 + \frac{f x}{e}\right] - 12 b^3 e^2 m n \log[x] \log[c x^n]^2 \log\left[1 + \frac{f x}{e}\right] - \\
& \quad 6 b e^2 m n \left( 2 a^2 - 2 a b n + b^2 n^2 - 2 b (-2 a + b n) \log[c x^n] + 2 b^2 \log[c x^n]^2 \right) \text{PolyLog}[2, -\frac{f x}{e}] + \\
& \quad 12 b^2 e^2 m n^2 \left( 2 a - b n + 2 b \log[c x^n] \right) \text{PolyLog}[3, -\frac{f x}{e}] - 24 b^3 e^2 m n^3 \text{PolyLog}[4, -\frac{f x}{e}]
\end{aligned}$$

**Problem 86: Result more than twice size of optimal antiderivative.**

$$\int (a + b \log[c x^n])^3 \log[d (e + f x)^m] dx$$

Optimal (type 4, 473 leaves, 28 steps):

$$\begin{aligned}
& -12 a b^2 m n^2 x + 18 b^3 m n^3 x - 6 b^2 m n^2 (a - b n) x - \\
& 18 b^3 m n^2 x \operatorname{Log}[c x^n] + 6 b m n x (a + b \operatorname{Log}[c x^n])^2 - m x (a + b \operatorname{Log}[c x^n])^3 + \\
& \frac{6 b^2 e m n^2 (a - b n) \operatorname{Log}[e + f x]}{f} + 6 a b^2 n^2 x \operatorname{Log}[d (e + f x)^m] - 6 b^3 n^3 x \operatorname{Log}[d (e + f x)^m] + \\
& 6 b^3 n^2 x \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] - 3 b n x (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m] + \\
& x (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m] + \frac{6 b^3 e m n^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{f} - \\
& \frac{3 b e m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{f} + \frac{e m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{f} + \\
& \frac{6 b^3 e m n^3 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{f} - \frac{6 b^2 e m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{f} + \\
& \frac{3 b e m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{f} + \frac{6 b^3 e m n^3 \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{f} - \\
& \frac{6 b^2 e m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{f} + \frac{6 b^3 e m n^3 \operatorname{PolyLog}\left[4, -\frac{f x}{e}\right]}{f}
\end{aligned}$$

Result (type 4, 1122 leaves):

$$\begin{aligned}
& \frac{1}{f} \left( -a^3 f m x + 6 a^2 b f m n x - 18 a b^2 f m n^2 x + 24 b^3 f m n^3 x - 3 a^2 b f m x \log[c x^n] + \right. \\
& \quad 12 a b^2 f m n x \log[c x^n] - 18 b^3 f m n^2 x \log[c x^n] - 3 a b^2 f m x \log[c x^n]^2 + 6 b^3 f m n x \log[c x^n]^2 - \\
& \quad b^3 f m x \log[c x^n]^3 + a^3 e m \log[e + f x] - 3 a^2 b e m n \log[e + f x] + 6 a b^2 e m n^2 \log[e + f x] - \\
& \quad 6 b^3 e m n^3 \log[e + f x] - 3 a^2 b e m n \log[x] \log[e + f x] + 6 a b^2 e m n^2 \log[x] \log[e + f x] - \\
& \quad 6 b^3 e m n^3 \log[x] \log[e + f x] + 3 a b^2 e m n^2 \log[x]^2 \log[e + f x] - 3 b^3 e m n^3 \log[x]^2 \log[e + f x] - \\
& \quad b^3 e m n^3 \log[x]^3 \log[e + f x] + 3 a^2 b e m \log[c x^n] \log[e + f x] - 6 a b^2 e m n \log[c x^n] \log[e + f x] + \\
& \quad 6 b^3 e m n^2 \log[c x^n] \log[e + f x] - 6 a b^2 e m n \log[x] \log[c x^n] \log[e + f x] + \\
& \quad 6 b^3 e m n^2 \log[x] \log[c x^n] \log[e + f x] + 3 b^3 e m n^2 \log[x]^2 \log[c x^n] \log[e + f x] + \\
& \quad 3 a b^2 e m \log[c x^n]^2 \log[e + f x] - 3 b^3 e m n \log[c x^n]^2 \log[e + f x] - \\
& \quad 3 b^3 e m n \log[x] \log[c x^n]^2 \log[e + f x] + b^3 e m \log[c x^n]^3 \log[e + f x] + a^3 f x \log[d (e + f x)^m] - \\
& \quad 3 a^2 b f n x \log[d (e + f x)^m] + 6 a b^2 f n^2 x \log[d (e + f x)^m] - 6 b^3 f n^3 x \log[d (e + f x)^m] + \\
& \quad 3 a^2 b f x \log[c x^n] \log[d (e + f x)^m] - 6 a b^2 f n x \log[c x^n] \log[d (e + f x)^m] + \\
& \quad 6 b^3 f n^2 x \log[c x^n] \log[d (e + f x)^m] + 3 a b^2 f x \log[c x^n]^2 \log[d (e + f x)^m] - \\
& \quad 3 b^3 f n x \log[c x^n]^2 \log[d (e + f x)^m] + b^3 f x \log[c x^n]^3 \log[d (e + f x)^m] + \\
& \quad 3 a^2 b e m n \log[x] \log[1 + \frac{f x}{e}] - 6 a b^2 e m n^2 \log[x] \log[1 + \frac{f x}{e}] + \\
& \quad 6 b^3 e m n^3 \log[x] \log[1 + \frac{f x}{e}] - 3 a b^2 e m n^2 \log[x]^2 \log[1 + \frac{f x}{e}] + \\
& \quad 3 b^3 e m n^3 \log[x]^2 \log[1 + \frac{f x}{e}] + b^3 e m n^3 \log[x]^3 \log[1 + \frac{f x}{e}] + \\
& \quad 6 a b^2 e m n \log[x] \log[c x^n] \log[1 + \frac{f x}{e}] - 6 b^3 e m n^2 \log[x] \log[c x^n] \log[1 + \frac{f x}{e}] - \\
& \quad 3 b^3 e m n^2 \log[x]^2 \log[c x^n] \log[1 + \frac{f x}{e}] + 3 b^3 e m n \log[x] \log[c x^n]^2 \log[1 + \frac{f x}{e}] + \\
& \quad 3 b e m n \left( a^2 - 2 a b n + 2 b^2 n^2 + 2 b (a - b n) \log[c x^n] + b^2 \log[c x^n]^2 \right) \text{PolyLog}[2, -\frac{f x}{e}] - \\
& \quad 6 b^2 e m n^2 \left( a - b n + b \log[c x^n] \right) \text{PolyLog}[3, -\frac{f x}{e}] + 6 b^3 e m n^3 \text{PolyLog}[4, -\frac{f x}{e}]
\end{aligned}$$

**Problem 87: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \log[c x^n])^3 \log[d (e + f x)^m]}{x} dx$$

Optimal (type 4, 161 leaves, 6 steps):

$$\begin{aligned}
& \frac{(a + b \log[c x^n])^4 \log[d (e + f x)^m]}{4 b n} - \frac{m (a + b \log[c x^n])^4 \log[1 + \frac{f x}{e}]}{4 b n} - \\
& m (a + b \log[c x^n])^3 \text{PolyLog}[2, -\frac{f x}{e}] + 3 b m n (a + b \log[c x^n])^2 \text{PolyLog}[3, -\frac{f x}{e}] - \\
& 6 b^2 m n^2 (a + b \log[c x^n]) \text{PolyLog}[4, -\frac{f x}{e}] + 6 b^3 m n^3 \text{PolyLog}[5, -\frac{f x}{e}]
\end{aligned}$$

Result (type 4, 602 leaves):

$$\begin{aligned}
& a^3 \operatorname{Log}[x] \operatorname{Log}[d (e + f x)^m] - \frac{3}{2} a^2 b n \operatorname{Log}[x]^2 \operatorname{Log}[d (e + f x)^m] + a b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[d (e + f x)^m] - \\
& \frac{1}{4} b^3 n^3 \operatorname{Log}[x]^4 \operatorname{Log}[d (e + f x)^m] + 3 a^2 b \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] - \\
& 3 a b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] + b^3 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x)^m] + \\
& 3 a b^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x)^m] - \frac{3}{2} b^3 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x)^m] + \\
& b^3 \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 \operatorname{Log}[d (e + f x)^m] - a^3 m \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{f x}{e}\right] + \\
& \frac{3}{2} a^2 b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right] - a b^2 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{f x}{e}\right] + \frac{1}{4} b^3 m n^3 \operatorname{Log}[x]^4 \operatorname{Log}\left[1 + \frac{f x}{e}\right] - \\
& 3 a^2 b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{f x}{e}\right] + 3 a b^2 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{f x}{e}\right] - \\
& b^3 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{f x}{e}\right] - 3 a b^2 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right] + \\
& \frac{3}{2} b^3 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right] - b^3 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 \operatorname{Log}\left[1 + \frac{f x}{e}\right] - \\
& m (a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}[2, -\frac{f x}{e}] + 3 b m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[3, -\frac{f x}{e}] - \\
& 6 a b^2 m n^2 \operatorname{PolyLog}[4, -\frac{f x}{e}] - 6 b^3 m n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[4, -\frac{f x}{e}] + 6 b^3 m n^3 \operatorname{PolyLog}[5, -\frac{f x}{e}]
\end{aligned}$$

**Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{x^2} dx$$

Optimal (type 4, 411 leaves, 14 steps):

$$\begin{aligned}
& \frac{6 b^3 f m n^3 \operatorname{Log}[x]}{e} - \frac{6 b^2 f m n^2 \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])}{e} - \\
& \frac{3 b f m n \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^2}{e} - \frac{f m \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^3}{e} - \\
& \frac{6 b^3 f m n^3 \operatorname{Log}[e + f x]}{e} - \frac{6 b^3 n^3 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{x} - \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{x} + \\
& \frac{6 b^3 f m n^3 \operatorname{PolyLog}[2, -\frac{e}{f x}]}{e} + \frac{6 b^2 f m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{e}{f x}]}{e} + \\
& \frac{3 b f m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{e}{f x}]}{e} + \frac{6 b^3 f m n^3 \operatorname{PolyLog}[3, -\frac{e}{f x}]}{e} + \\
& \frac{6 b^2 f m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{e}{f x}]}{e} + \frac{6 b^3 f m n^3 \operatorname{PolyLog}[4, -\frac{e}{f x}]}{e}
\end{aligned}$$

Result (type 4, 1347 leaves):

$$\begin{aligned}
& -\frac{1}{4 e x} \left( -4 a^3 f m x \log[x] - 12 a^2 b f m n x \log[x] - 24 a b^2 f m n^2 x \log[x] - \right. \\
& \quad 24 b^3 f m n^3 x \log[x] + 6 a^2 b f m n x \log[x]^2 + 12 a b^2 f m n^2 x \log[x]^2 + \\
& \quad 12 b^3 f m n^3 x \log[x]^2 - 4 a b^2 f m n^2 x \log[x]^3 - 4 b^3 f m n^3 x \log[x]^3 + \\
& \quad b^3 f m n^3 x \log[x]^4 - 12 a^2 b f m x \log[x] \log[c x^n] - 24 a b^2 f m n x \log[x] \log[c x^n] - \\
& \quad 24 b^3 f m n^2 x \log[x] \log[c x^n] + 12 a b^2 f m n x \log[x]^2 \log[c x^n] + \\
& \quad 12 b^3 f m n^2 x \log[x]^2 \log[c x^n] - 4 b^3 f m n^2 x \log[x]^3 \log[c x^n] - \\
& \quad 12 a b^2 f m x \log[x] \log[c x^n]^2 - 12 b^3 f m n x \log[x] \log[c x^n]^2 + 6 b^3 f m n x \log[x]^2 \log[c x^n]^2 - \\
& \quad 4 b^3 f m x \log[x] \log[c x^n]^3 + 4 a^3 f m x \log[e + f x] + 12 a^2 b f m n x \log[e + f x] + \\
& \quad 24 a b^2 f m n^2 x \log[e + f x] + 24 b^3 f m n^3 x \log[e + f x] - 12 a^2 b f m n x \log[x] \log[e + f x] - \\
& \quad 24 a b^2 f m n^2 x \log[x] \log[e + f x] - 24 b^3 f m n^3 x \log[x] \log[e + f x] + \\
& \quad 12 a b^2 f m n^2 x \log[x]^2 \log[e + f x] + 12 b^3 f m n^3 x \log[x]^2 \log[e + f x] - \\
& \quad 4 b^3 f m n^3 x \log[x]^3 \log[e + f x] + 12 a^2 b f m x \log[c x^n] \log[e + f x] + \\
& \quad 24 a b^2 f m n x \log[c x^n] \log[e + f x] + 24 b^3 f m n^2 x \log[c x^n] \log[e + f x] - \\
& \quad 24 a b^2 f m n x \log[x] \log[c x^n] \log[e + f x] - 24 b^3 f m n^2 x \log[x] \log[c x^n] \log[e + f x] + \\
& \quad 12 b^3 f m n^2 x \log[x]^2 \log[c x^n] \log[e + f x] + 12 a b^2 f m x \log[c x^n]^2 \log[e + f x] + \\
& \quad 12 b^3 f m n x \log[c x^n]^2 \log[e + f x] - 12 b^3 f m n x \log[x] \log[c x^n]^2 \log[e + f x] + \\
& \quad 4 b^3 f m x \log[c x^n]^3 \log[e + f x] + 4 a^3 e \log[d (e + f x)^m] + \\
& \quad 12 a^2 b e n \log[d (e + f x)^m] + 24 a b^2 e n^2 \log[d (e + f x)^m] + 24 b^3 e n^3 \log[d (e + f x)^m] + \\
& \quad 12 a^2 b e \log[c x^n] \log[d (e + f x)^m] + 24 a b^2 e n \log[c x^n] \log[d (e + f x)^m] + \\
& \quad 24 b^3 e n^2 \log[c x^n] \log[d (e + f x)^m] + 12 a b^2 e \log[c x^n]^2 \log[d (e + f x)^m] + \\
& \quad 12 b^3 e n \log[c x^n]^2 \log[d (e + f x)^m] + 4 b^3 e \log[c x^n]^3 \log[d (e + f x)^m] + \\
& \quad 12 a^2 b f m n x \log[x] \log[1 + \frac{f x}{e}] + 24 a b^2 f m n^2 x \log[x] \log[1 + \frac{f x}{e}] + \\
& \quad 24 b^3 f m n^3 x \log[x] \log[1 + \frac{f x}{e}] - 12 a b^2 f m n^2 x \log[x]^2 \log[1 + \frac{f x}{e}] - \\
& \quad 12 b^3 f m n^3 x \log[x]^2 \log[1 + \frac{f x}{e}] + 4 b^3 f m n^3 x \log[x]^3 \log[1 + \frac{f x}{e}] + \\
& \quad 24 a b^2 f m n x \log[x] \log[c x^n] \log[1 + \frac{f x}{e}] + 24 b^3 f m n^2 x \log[x] \log[c x^n] \log[1 + \frac{f x}{e}] - \\
& \quad 12 b^3 f m n^2 x \log[x]^2 \log[c x^n] \log[1 + \frac{f x}{e}] + 12 b^3 f m n x \log[x] \log[c x^n]^2 \log[1 + \frac{f x}{e}] + \\
& \quad 12 b f m n x \left( a^2 + 2 a b n + 2 b^2 n^2 + 2 b (a + b n) \log[c x^n] + b^2 \log[c x^n]^2 \right) \text{PolyLog}[2, -\frac{f x}{e}] - \\
& \quad 24 b^2 f m n^2 x \left( a + b n + b \log[c x^n] \right) \text{PolyLog}[3, -\frac{f x}{e}] + 24 b^3 f m n^3 x \text{PolyLog}[4, -\frac{f x}{e}]
\end{aligned}$$

**Problem 89: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \log[c x^n])^3 \log[d (e + f x)^m]}{x^3} dx$$

Optimal (type 4, 555 leaves, 22 steps):

$$\begin{aligned}
& - \frac{45 b^3 f m n^3}{8 e x} - \frac{3 b^3 f^2 m n^3 \log[x]}{8 e^2} - \frac{21 b^2 f m n^2 (a + b \log[c x^n])}{4 e x} + \\
& \frac{3 b^2 f^2 m n^2 \log[1 + \frac{e}{f x}] (a + b \log[c x^n])}{4 e^2} - \frac{9 b f m n (a + b \log[c x^n])^2}{4 e x} + \\
& \frac{3 b f^2 m n \log[1 + \frac{e}{f x}] (a + b \log[c x^n])^2}{4 e^2} - \frac{f m (a + b \log[c x^n])^3}{2 e x} + \\
& \frac{f^2 m \log[1 + \frac{e}{f x}] (a + b \log[c x^n])^3}{2 e^2} + \frac{3 b^3 f^2 m n^3 \log[e + f x]}{8 e^2} - \\
& \frac{3 b^3 n^3 \log[d (e + f x)^m]}{8 x^2} - \frac{3 b^2 n^2 (a + b \log[c x^n]) \log[d (e + f x)^m]}{4 x^2} - \\
& \frac{3 b n (a + b \log[c x^n])^2 \log[d (e + f x)^m]}{4 x^2} - \frac{(a + b \log[c x^n])^3 \log[d (e + f x)^m]}{2 x^2} - \\
& \frac{3 b^3 f^2 m n^3 \text{PolyLog}[2, -\frac{e}{f x}]}{4 e^2} - \frac{3 b^2 f^2 m n^2 (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{e}{f x}]}{2 e^2} - \\
& \frac{3 b f^2 m n (a + b \log[c x^n])^2 \text{PolyLog}[2, -\frac{e}{f x}]}{2 e^2} - \frac{3 b^3 f^2 m n^3 \text{PolyLog}[3, -\frac{e}{f x}]}{2 e^2} - \\
& \frac{3 b^2 f^2 m n^2 (a + b \log[c x^n]) \text{PolyLog}[3, -\frac{e}{f x}]}{e^2} - \frac{3 b^3 f^2 m n^3 \text{PolyLog}[4, -\frac{e}{f x}]}{e^2}
\end{aligned}$$

Result (type 4, 1736 leaves) :

$$\begin{aligned}
& - \frac{1}{8 e^2 x^2} \left( 4 a^3 e f m x + 18 a^2 b e f m n x + 42 a b^2 e f m n^2 x + 45 b^3 e f m n^3 x + 4 a^3 f^2 m x^2 \log[x] + \right. \\
& \quad 6 a^2 b f^2 m n x^2 \log[x] + 6 a b^2 f^2 m n^2 x^2 \log[x] + 3 b^3 f^2 m n^3 x^2 \log[x] - 6 a^2 b f^2 m n x^2 \log[x]^2 - \\
& \quad 6 a b^2 f^2 m n^2 x^2 \log[x]^2 - 3 b^3 f^2 m n^3 x^2 \log[x]^2 + 4 a b^2 f^2 m n^2 x^2 \log[x]^3 + 2 b^3 f^2 m n^3 x^2 \log[x]^3 - \\
& \quad b^3 f^2 m n^3 x^2 \log[x]^4 + 12 a^2 b e f m x \log[c x^n] + 36 a b^2 e f m n x \log[c x^n] + \\
& \quad 42 b^3 e f m n^2 x \log[c x^n] + 12 a^2 b f^2 m x^2 \log[x] \log[c x^n] + 12 a b^2 f^2 m n x^2 \log[x] \log[c x^n] + \\
& \quad 6 b^3 f^2 m n^2 x^2 \log[x] \log[c x^n] - 12 a b^2 f^2 m n x^2 \log[x]^2 \log[c x^n] - \\
& \quad 6 b^3 f^2 m n^2 x^2 \log[x]^2 \log[c x^n] + 4 b^3 f^2 m n^2 x^2 \log[x]^3 \log[c x^n] + \\
& \quad 12 a b^2 e f m x \log[c x^n]^2 + 18 b^3 e f m n x \log[c x^n]^2 + 12 a b^2 f^2 m x^2 \log[x] \log[c x^n]^2 + \\
& \quad 6 b^3 f^2 m n x^2 \log[x] \log[c x^n]^2 - 6 b^3 f^2 m n x^2 \log[x]^2 \log[c x^n]^2 + 4 b^3 e f m x \log[c x^n]^3 + \\
& \quad 4 b^3 f^2 m x^2 \log[x] \log[c x^n]^3 - 4 a^3 f^2 m x^2 \log[e + f x] - 6 a^2 b f^2 m n x^2 \log[e + f x] - \\
& \quad 6 a b^2 f^2 m n^2 x^2 \log[e + f x] - 3 b^3 f^2 m n^3 x^2 \log[e + f x] + 12 a^2 b f^2 m n x^2 \log[e + f x] \log[e + f x] + \\
& \quad 12 a b^2 f^2 m n^2 x^2 \log[x] \log[e + f x] + 6 b^3 f^2 m n^3 x^2 \log[x] \log[e + f x] - \\
& \quad 12 a b^2 f^2 m n^2 x^2 \log[e + f x]^2 - 6 b^3 f^2 m n^3 x^2 \log[x]^2 \log[e + f x] + \\
& \quad 4 b^3 f^2 m n^3 x^2 \log[x]^3 \log[e + f x] - 12 a^2 b f^2 m x^2 \log[c x^n] \log[e + f x] - \\
& \quad 12 a b^2 f^2 m n x^2 \log[c x^n] \log[e + f x] - 6 b^3 f^2 m n^2 x^2 \log[c x^n] \log[e + f x] + \\
& \quad 24 a b^2 f^2 m n x^2 \log[x] \log[c x^n] \log[e + f x] + 12 b^3 f^2 m n^2 x^2 \log[x] \log[c x^n] \log[e + f x] - \\
& \quad 12 b^3 f^2 m n^2 x^2 \log[x]^2 \log[c x^n] \log[e + f x] - 12 a b^2 f^2 m x^2 \log[c x^n]^2 \log[e + f x] - \\
& \quad 6 b^3 f^2 m n x^2 \log[c x^n]^2 \log[e + f x] + 12 b^3 f^2 m n x^2 \log[x] \log[c x^n]^2 \log[e + f x] - \\
& \quad 4 b^3 f^2 m x^2 \log[c x^n]^3 \log[e + f x] + 4 a^3 e^2 \log[d (e + f x)^m] + \\
& \quad 6 a^2 b e^2 n \log[d (e + f x)^m] + 6 a b^2 e^2 n^2 \log[d (e + f x)^m] + 3 b^3 e^2 n^3 \log[d (e + f x)^m] + \\
& \quad 12 a^2 b e^2 \log[c x^n] \log[d (e + f x)^m] + 12 a b^2 e^2 n \log[c x^n] \log[d (e + f x)^m] + \\
& \quad 6 b^3 e^2 n^2 \log[c x^n] \log[d (e + f x)^m] + 12 a b^2 e^2 \log[c x^n]^2 \log[d (e + f x)^m] + \\
& \quad 6 b^3 e^2 n \log[c x^n]^2 \log[d (e + f x)^m] + 4 b^3 e^2 \log[c x^n]^3 \log[d (e + f x)^m] - \\
& \quad 12 a^2 b f^2 m n x^2 \log[x] \log[1 + \frac{f x}{e}] - 12 a b^2 f^2 m n^2 x^2 \log[x] \log[1 + \frac{f x}{e}] - \\
& \quad 6 b^3 f^2 m n^3 x^2 \log[x] \log[1 + \frac{f x}{e}] + 12 a b^2 f^2 m n^2 x^2 \log[x]^2 \log[1 + \frac{f x}{e}] + \\
& \quad 6 b^3 f^2 m n^3 x^2 \log[x]^2 \log[1 + \frac{f x}{e}] - 4 b^3 f^2 m n^3 x^2 \log[x]^3 \log[1 + \frac{f x}{e}] - \\
& \quad 24 a b^2 f^2 m n x^2 \log[x] \log[c x^n] \log[1 + \frac{f x}{e}] - 12 b^3 f^2 m n^2 x^2 \log[x] \log[c x^n] \log[1 + \frac{f x}{e}] + \\
& \quad 12 b^3 f^2 m n^2 x^2 \log[x]^2 \log[c x^n] \log[1 + \frac{f x}{e}] - 12 b^3 f^2 m n x^2 \log[x] \log[c x^n]^2 \log[1 + \frac{f x}{e}] - \\
& \quad 6 b f^2 m n x^2 (2 a^2 + 2 a b n + b^2 n^2 + 2 b (2 a + b n) \log[c x^n] + 2 b^2 \log[c x^n]^2) \text{PolyLog}[2, -\frac{f x}{e}] + \\
& \quad \left. 12 b^2 f^2 m n^2 x^2 (2 a + b n + 2 b \log[c x^n]) \text{PolyLog}[3, -\frac{f x}{e}] - 24 b^3 f^2 m n^3 x^2 \text{PolyLog}[4, -\frac{f x}{e}] \right)
\end{aligned}$$

**Problem 90: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3 (a + b \log[c x^n]) \log[d (e + f x^2)^m] dx$$

Optimal (type 4, 221 leaves, 9 steps):

$$\begin{aligned}
& -\frac{3 b e m n x^2}{16 f} + \frac{1}{16} b m n x^4 + \frac{e m x^2 (a + b \log[c x^n])}{4 f} - \frac{1}{8} m x^4 (a + b \log[c x^n]) + \\
& \frac{b e^2 m n \log[e + f x^2]}{16 f^2} + \frac{b e^2 m n \log[-\frac{f x^2}{e}] \log[e + f x^2]}{8 f^2} - \frac{e^2 m (a + b \log[c x^n]) \log[e + f x^2]}{4 f^2} - \\
& \frac{1}{16} b n x^4 \log[d (e + f x^2)^m] + \frac{1}{4} x^4 (a + b \log[c x^n]) \log[d (e + f x^2)^m] + \frac{b e^2 m n \text{PolyLog}[2, 1 + \frac{f x^2}{e}]}{8 f^2}
\end{aligned}$$

Result (type 4, 324 leaves):

$$\begin{aligned}
& -\frac{1}{16 f^2} \\
& \left( -4 a e f m x^2 + 3 b e f m n x^2 + 2 a f^2 m x^4 - b f^2 m n x^4 - 4 b e f m x^2 \log[c x^n] + 2 b f^2 m x^4 \log[c x^n] + \right. \\
& 4 b e^2 m n \log[x] \log[1 - \frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] + 4 b e^2 m n \log[x] \log[1 + \frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] + 4 a e^2 m \log[e + f x^2] - \\
& b e^2 m n \log[e + f x^2] - 4 b e^2 m n \log[x] \log[e + f x^2] + 4 b e^2 m \log[c x^n] \log[e + f x^2] - \\
& 4 a f^2 x^4 \log[d (e + f x^2)^m] + b f^2 n x^4 \log[d (e + f x^2)^m] - 4 b f^2 x^4 \log[c x^n] \log[d (e + f x^2)^m] + \\
& \left. 4 b e^2 m n \text{PolyLog}[2, -\frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] + 4 b e^2 m n \text{PolyLog}[2, \frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] \right)
\end{aligned}$$

**Problem 91: Result unnecessarily involves imaginary or complex numbers.**

$$\int x (a + b \log[c x^n]) \log[d (e + f x^2)^m] dx$$

Optimal (type 4, 148 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{2} b m n x^2 - \frac{1}{2} m x^2 (a + b \log[c x^n]) - \\
& \frac{b n (e + f x^2) \log[d (e + f x^2)^m]}{4 f} - \frac{b e n \log[-\frac{f x^2}{e}] \log[d (e + f x^2)^m]}{4 f} + \\
& \frac{(e + f x^2) (a + b \log[c x^n]) \log[d (e + f x^2)^m]}{2 f} - \frac{b e m n \text{PolyLog}[2, 1 + \frac{f x^2}{e}]}{4 f}
\end{aligned}$$

Result (type 4, 263 leaves):

$$\begin{aligned}
& \frac{1}{4 f} \left( -2 a f m x^2 + 2 b f m n x^2 - 2 b f m x^2 \log[c x^n] + 2 b e m n \log[x] \log[1 - \frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] + \right. \\
& 2 b e m n \log[x] \log[1 + \frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] + 2 a e m \log[e + f x^2] - b e m n \log[e + f x^2] - \\
& 2 b e m n \log[x] \log[e + f x^2] + 2 b e m \log[c x^n] \log[e + f x^2] + 2 a f x^2 \log[d (e + f x^2)^m] - \\
& b f n x^2 \log[d (e + f x^2)^m] + 2 b f x^2 \log[c x^n] \log[d (e + f x^2)^m] + \\
& \left. 2 b e m n \text{PolyLog}[2, -\frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] + 2 b e m n \text{PolyLog}[2, \frac{\frac{i}{2} \sqrt{f} x}{\sqrt{e}}] \right)
\end{aligned}$$

**Problem 92:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{x} dx$$

Optimal (type 4, 113 leaves, 4 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{2 b n} - \frac{m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x^2}{e}\right]}{2 b n} - \\ & \frac{1}{2} m (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x^2}{e}\right] + \frac{1}{4} b m n \operatorname{PolyLog}\left[3, -\frac{f x^2}{e}\right] \end{aligned}$$

Result (type 4, 307 leaves):

$$\begin{aligned} & \frac{1}{2} \left( b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 2 b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \right. \\ & b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 2 b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\ & 2 a \operatorname{Log}[x] \operatorname{Log}[d (e + f x^2)^m] - b n \operatorname{Log}[x]^2 \operatorname{Log}[d (e + f x^2)^m] + \\ & 2 b \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - 2 a m \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{f x^2}{e}\right] - \\ & 2 b m \operatorname{Log}[c x^n] \operatorname{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right] - 2 b m \operatorname{Log}[c x^n] \operatorname{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\ & a m \operatorname{PolyLog}\left[2, -\frac{f x^2}{e}\right] + 2 b m n \operatorname{PolyLog}\left[3, -\frac{i \sqrt{f} x}{\sqrt{e}}\right] + 2 b m n \operatorname{PolyLog}\left[3, \frac{i \sqrt{f} x}{\sqrt{e}}\right] \left. \right) \end{aligned}$$

**Problem 93:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{x^3} dx$$

Optimal (type 4, 195 leaves, 11 steps):

$$\begin{aligned} & \frac{b f m n \operatorname{Log}[x]}{2 e} - \frac{b f m n \operatorname{Log}[x]^2}{2 e} + \frac{f m \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])}{e} - \frac{b f m n \operatorname{Log}[e + f x^2]}{4 e} + \\ & \frac{b f m n \operatorname{Log}\left[-\frac{f x^2}{e}\right] \operatorname{Log}[e + f x^2]}{4 e} - \frac{f m (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[e + f x^2]}{2 e} - \\ & \frac{b n \operatorname{Log}[d (e + f x^2)^m]}{4 x^2} - \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{2 x^2} + \frac{b f m n \operatorname{PolyLog}\left[2, 1 + \frac{f x^2}{e}\right]}{4 e} \end{aligned}$$

Result (type 4, 298 leaves):

$$\begin{aligned}
& -\frac{1}{4 e x^2} \\
& \left( -4 a f m x^2 \operatorname{Log}[x] - 2 b f m n x^2 \operatorname{Log}[x] + 2 b f m n x^2 \operatorname{Log}[x]^2 - 4 b f m x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] + 2 b f m n \right. \\
& \quad x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}\right] + 2 b f m n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}\right] + 2 a f m x^2 \operatorname{Log}[e + f x^2] + \\
& \quad b f m n x^2 \operatorname{Log}[e + f x^2] - 2 b f m n x^2 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] + 2 b f m x^2 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + \\
& \quad 2 a e \operatorname{Log}[d (e + f x^2)^m] + b e n \operatorname{Log}[d (e + f x^2)^m] + 2 b e \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + \\
& \quad \left. 2 b f m n x^2 \operatorname{PolyLog}[2, -\frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}] + 2 b f m n x^2 \operatorname{PolyLog}[2, \frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}] \right)
\end{aligned}$$

**Problem 94:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{x^5} dx$$

Optimal (type 4, 248 leaves, 10 steps):

$$\begin{aligned}
& -\frac{3 b f m n}{16 e x^2} - \frac{b f^2 m n \operatorname{Log}[x]}{8 e^2} + \frac{b f^2 m n \operatorname{Log}[x]^2}{4 e^2} - \frac{f m (a + b \operatorname{Log}[c x^n])}{4 e x^2} - \frac{f^2 m \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])}{2 e^2} + \\
& \frac{b f^2 m n \operatorname{Log}[e + f x^2]}{16 e^2} - \frac{b f^2 m n \operatorname{Log}[-\frac{f x^2}{e}] \operatorname{Log}[e + f x^2]}{8 e^2} + \frac{f^2 m (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[e + f x^2]}{4 e^2} - \\
& \frac{b n \operatorname{Log}[d (e + f x^2)^m]}{16 x^4} - \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{4 x^4} - \frac{b f^2 m n \operatorname{PolyLog}[2, 1 + \frac{f x^2}{e}]}{8 e^2}
\end{aligned}$$

Result (type 4, 363 leaves):

$$\begin{aligned}
& -\frac{1}{16 e^2 x^4} \left( 4 a e f m x^2 + 3 b e f m n x^2 + 8 a f^2 m x^4 \operatorname{Log}[x] + 2 b f^2 m n x^4 \operatorname{Log}[x] - 4 b f^2 m n x^4 \operatorname{Log}[x]^2 + \right. \\
& \quad 4 b e f m x^2 \operatorname{Log}[c x^n] + 8 b f^2 m x^4 \operatorname{Log}[x] \operatorname{Log}[c x^n] - 4 b f^2 m n x^4 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}\right] - \\
& \quad 4 b f^2 m n x^4 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}\right] - 4 a f^2 m x^4 \operatorname{Log}[e + f x^2] - b f^2 m n x^4 \operatorname{Log}[e + f x^2] + \\
& \quad 4 b f^2 m n x^4 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] - 4 b f^2 m x^4 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + \\
& \quad 4 a e^2 \operatorname{Log}[d (e + f x^2)^m] + b e^2 n \operatorname{Log}[d (e + f x^2)^m] + 4 b e^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - \\
& \quad \left. 4 b f^2 m n x^4 \operatorname{PolyLog}[2, -\frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}] - 4 b f^2 m n x^4 \operatorname{PolyLog}[2, \frac{\frac{i \sqrt{f} x}{\sqrt{e}}}{\sqrt{e}}] \right)
\end{aligned}$$

**Problem 100:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m] dx$$

Optimal (type 4, 310 leaves, 17 steps):

$$\begin{aligned}
 & -\frac{3}{4} b^2 m n^2 x^2 + b m n x^2 (a + b \operatorname{Log}[c x^n]) - \frac{1}{2} m x^2 (a + b \operatorname{Log}[c x^n])^2 + \frac{b^2 e m n^2 \operatorname{Log}[e + f x^2]}{4 f} + \\
 & \frac{1}{4} b^2 n^2 x^2 \operatorname{Log}[d (e + f x^2)^m] - \frac{1}{2} b n x^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m] + \\
 & \frac{1}{2} x^2 (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m] - \frac{b e m n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + \frac{f x^2}{e}]}{2 f} + \\
 & \frac{e m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + \frac{f x^2}{e}]}{2 f} - \frac{b^2 e m n^2 \operatorname{PolyLog}[2, -\frac{f x^2}{e}]}{4 f} + \\
 & \frac{b e m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{f x^2}{e}]}{2 f} - \frac{b^2 e m n^2 \operatorname{PolyLog}[3, -\frac{f x^2}{e}]}{4 f}
 \end{aligned}$$

Result (type 4, 814 leaves):

$$\begin{aligned}
 & \frac{1}{4 f} \left( -2 a^2 f m x^2 + 4 a b f m n x^2 - 3 b^2 f m n^2 x^2 - 4 a b f m x^2 \operatorname{Log}[c x^n] + 4 b^2 f m n x^2 \operatorname{Log}[c x^n] - \right. \\
 & 2 b^2 f m x^2 \operatorname{Log}[c x^n]^2 + 4 a b e m n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 2 b^2 e m n^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
 & 2 b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 4 b^2 e m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
 & 4 a b e m n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 2 b^2 e m n^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
 & 2 b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 4 b^2 e m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
 & 2 a^2 e m \operatorname{Log}[e + f x^2] - 2 a b e m n \operatorname{Log}[e + f x^2] + b^2 e m n^2 \operatorname{Log}[e + f x^2] - \\
 & 4 a b e m n \operatorname{Log}[x] \operatorname{Log}[e + f x^2] + 2 b^2 e m n^2 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] + 2 b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[e + f x^2] + \\
 & 4 a b e m \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] - 2 b^2 e m n \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] - \\
 & 4 b^2 e m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + 2 b^2 e m \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x^2] + \\
 & 2 a^2 f x^2 \operatorname{Log}[d (e + f x^2)^m] - 2 a b f n x^2 \operatorname{Log}[d (e + f x^2)^m] + b^2 f n^2 x^2 \operatorname{Log}[d (e + f x^2)^m] + \\
 & 4 a b f x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - 2 b^2 f n x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + \\
 & 2 b^2 f x^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] + 2 b e m n (2 a - b n + 2 b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}] + \\
 & 2 b e m n (2 a - b n + 2 b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
 & \left. 4 b^2 e m n^2 \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 4 b^2 e m n^2 \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] \right)
 \end{aligned}$$

**Problem 101:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{x} dx$$

Optimal (type 4, 147 leaves, 5 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m]}{3 b n} - \\ & \frac{m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{f x^2}{e}\right]}{3 b n} - \frac{1}{2} m (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{f x^2}{e}] + \\ & \frac{1}{2} b m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{f x^2}{e}] - \frac{1}{4} b^2 m n^2 \operatorname{PolyLog}[4, -\frac{f x^2}{e}] \end{aligned}$$

Result (type 4, 736 leaves):

$$\begin{aligned} & -a^2 m \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + a b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\ & \frac{1}{3} b^2 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 2 a b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\ & b^2 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - b^2 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\ & a^2 m \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + a b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\ & \frac{1}{3} b^2 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 2 a b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\ & b^2 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - b^2 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\ & a^2 \operatorname{Log}[x] \operatorname{Log}[d (e + f x^2)^m] - a b n \operatorname{Log}[x]^2 \operatorname{Log}[d (e + f x^2)^m] + \\ & \frac{1}{3} b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[d (e + f x^2)^m] + 2 a b \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - \\ & b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + b^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] - \\ & m (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}] - m (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\ & 2 a b m n \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 2 b^2 m n \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + \\ & 2 a b m n \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + 2 b^2 m n \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] - \\ & 2 b^2 m n^2 \operatorname{PolyLog}[4, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 2 b^2 m n^2 \operatorname{PolyLog}[4, \frac{i \sqrt{f} x}{\sqrt{e}}] \end{aligned}$$

Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{x^3} dx$$

Optimal (type 4, 276 leaves, 11 steps):

$$\begin{aligned} & \frac{b^2 f m n^2 \operatorname{Log}[x]}{2 e} - \frac{b f m n \operatorname{Log}\left[1 + \frac{e}{f x^2}\right] (a + b \operatorname{Log}[c x^n])}{2 e} - \frac{f m \operatorname{Log}\left[1 + \frac{e}{f x^2}\right] (a + b \operatorname{Log}[c x^n])^2}{2 e} - \\ & \frac{b^2 f m n^2 \operatorname{Log}[e + f x^2]}{4 e} - \frac{b^2 n^2 \operatorname{Log}[d (e + f x^2)^m]}{4 x^2} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{2 x^2} - \\ & \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{2 x^2} + \frac{b^2 f m n^2 \operatorname{PolyLog}[2, -\frac{e}{f x^2}]}{4 e} + \\ & \frac{b f m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{e}{f x^2}]}{2 e} + \frac{b^2 f m n^2 \operatorname{PolyLog}[3, -\frac{e}{f x^2}]}{4 e} \end{aligned}$$

Result (type 4, 946 leaves):

$$\begin{aligned}
& -\frac{1}{12 e x^2} \left( -12 a^2 f m n x^2 \log[x] - 12 a b f m n x^2 \log[x] - 6 b^2 f m n^2 x^2 \log[x] + 12 a b f m n x^2 \log[x]^2 + \right. \\
& \quad 6 b^2 f m n^2 x^2 \log[x]^2 - 4 b^2 f m n^2 x^2 \log[x]^3 - 24 a b f m n x^2 \log[x] \log[c x^n] - \\
& \quad 12 b^2 f m n x^2 \log[x] \log[c x^n] + 12 b^2 f m n x^2 \log[x]^2 \log[c x^n] - 12 b^2 f m n x^2 \log[x] \log[c x^n]^2 + \\
& \quad 12 a b f m n x^2 \log[x] \log\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 b^2 f m n^2 x^2 \log[x] \log\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& \quad 6 b^2 f m n^2 x^2 \log[x]^2 \log\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 12 b^2 f m n x^2 \log[x] \log[c x^n] \log\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& \quad 12 a b f m n x^2 \log[x] \log\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 b^2 f m n^2 x^2 \log[x] \log\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& \quad 6 b^2 f m n^2 x^2 \log[x]^2 \log\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 12 b^2 f m n x^2 \log[x] \log[c x^n] \log\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& \quad 6 a^2 f m n x^2 \log[e + f x^2] + 6 a b f m n x^2 \log[e + f x^2] + 3 b^2 f m n^2 x^2 \log[e + f x^2] - \\
& \quad 12 a b f m n x^2 \log[x] \log[e + f x^2] - 6 b^2 f m n^2 x^2 \log[x] \log[e + f x^2] + \\
& \quad 6 b^2 f m n^2 x^2 \log[x]^2 \log[e + f x^2] + 12 a b f m n x^2 \log[c x^n] \log[e + f x^2] + \\
& \quad 6 b^2 f m n x^2 \log[c x^n] \log[e + f x^2] - 12 b^2 f m n x^2 \log[x] \log[c x^n] \log[e + f x^2] + \\
& \quad 6 b^2 f m n x^2 \log[c x^n]^2 \log[e + f x^2] + 6 a^2 e \log[d (e + f x^2)^m] + 6 a b e n \log[d (e + f x^2)^m] + \\
& \quad 3 b^2 e n^2 \log[d (e + f x^2)^m] + 12 a b e \log[c x^n] \log[d (e + f x^2)^m] + \\
& \quad 6 b^2 e n \log[c x^n] \log[d (e + f x^2)^m] + 6 b^2 e \log[c x^n]^2 \log[d (e + f x^2)^m] + \\
& \quad 6 b f m n x^2 (2 a + b n + 2 b \log[c x^n]) \text{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& \quad 6 b f m n x^2 (2 a + b n + 2 b \log[c x^n]) \text{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& \quad \left. 12 b^2 f m n^2 x^2 \text{PolyLog}\left[3, -\frac{i \sqrt{f} x}{\sqrt{e}}\right] - 12 b^2 f m n^2 x^2 \text{PolyLog}\left[3, \frac{i \sqrt{f} x}{\sqrt{e}}\right] \right)
\end{aligned}$$

**Problem 103:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \log[c x^n])^2 \log[d (e + f x^2)^m]}{x^5} dx$$

Optimal (type 4, 356 leaves, 15 steps):

$$\begin{aligned}
& - \frac{7 b^2 f m n^2}{32 e x^2} - \frac{b^2 f^2 m n^2 \log[x]}{16 e^2} - \frac{3 b f m n (a + b \log[c x^n])}{8 e x^2} + \frac{b f^2 m n \log[1 + \frac{e}{f x^2}] (a + b \log[c x^n])}{8 e^2} - \\
& \frac{f m (a + b \log[c x^n])^2}{4 e x^2} + \frac{f^2 m \log[1 + \frac{e}{f x^2}] (a + b \log[c x^n])^2}{4 e^2} + \frac{b^2 f^2 m n^2 \log[e + f x^2]}{32 e^2} - \\
& \frac{b^2 n^2 \log[d (e + f x^2)^m]}{32 x^4} - \frac{b n (a + b \log[c x^n]) \log[d (e + f x^2)^m]}{8 x^4} - \\
& \frac{(a + b \log[c x^n])^2 \log[d (e + f x^2)^m]}{4 x^4} - \frac{b^2 f^2 m n^2 \text{PolyLog}[2, -\frac{e}{f x^2}]}{16 e^2} - \\
& \frac{b f^2 m n (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{e}{f x^2}]}{4 e^2} - \frac{b^2 f^2 m n^2 \text{PolyLog}[3, -\frac{e}{f x^2}]}{8 e^2}
\end{aligned}$$

Result (type 4, 1111 leaves) :

$$\begin{aligned}
& - \frac{1}{96 e^2 x^4} \\
& \left( 24 a^2 e f m x^2 + 36 a b e f m n x^2 + 21 b^2 e f m n^2 x^2 + 48 a^2 f^2 m x^4 \log[x] + 24 a b f^2 m n x^4 \log[x] + \right. \\
& \quad 6 b^2 f^2 m n^2 x^4 \log[x] - 48 a b f^2 m n x^4 \log[x]^2 - 12 b^2 f^2 m n^2 x^4 \log[x]^2 + 16 b^2 f^2 m n^2 x^4 \log[x]^3 + \\
& \quad 48 a b e f m x^2 \log[c x^n] + 36 b^2 e f m n x^2 \log[c x^n] + 96 a b f^2 m x^4 \log[x] \log[c x^n] + \\
& \quad 24 b^2 f^2 m n x^4 \log[x] \log[c x^n] - 48 b^2 f^2 m n x^4 \log[x]^2 \log[c x^n] + 24 b^2 e f m x^2 \log[c x^n]^2 + \\
& \quad 48 b^2 f^2 m x^4 \log[x] \log[c x^n]^2 - 48 a b f^2 m n x^4 \log[x] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& \quad 12 b^2 f^2 m n^2 x^4 \log[x] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + 24 b^2 f^2 m n^2 x^4 \log[x]^2 \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& \quad 48 b^2 f^2 m n x^4 \log[x] \log[c x^n] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] - 48 a b f^2 m n x^4 \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& \quad 12 b^2 f^2 m n^2 x^4 \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + 24 b^2 f^2 m n^2 x^4 \log[x]^2 \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& \quad 48 b^2 f^2 m n x^4 \log[x] \log[c x^n] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - 24 a^2 f^2 m x^4 \log[e + f x^2] - \\
& \quad 12 a b f^2 m n x^4 \log[e + f x^2] - 3 b^2 f^2 m n^2 x^4 \log[e + f x^2] + 48 a b f^2 m n x^4 \log[x] \log[e + f x^2] + \\
& \quad 12 b^2 f^2 m n^2 x^4 \log[x] \log[e + f x^2] - 24 b^2 f^2 m n^2 x^4 \log[x]^2 \log[e + f x^2] - \\
& \quad 48 a b f^2 m n x^4 \log[c x^n] \log[e + f x^2] - 12 b^2 f^2 m n x^4 \log[c x^n] \log[e + f x^2] + \\
& \quad 48 b^2 f^2 m n x^4 \log[x] \log[c x^n] \log[e + f x^2] - 24 b^2 f^2 m n x^4 \log[c x^n]^2 \log[e + f x^2] + \\
& \quad 24 a^2 e^2 \log[d (e + f x^2)^m] + 12 a b e^2 n \log[d (e + f x^2)^m] + \\
& \quad 3 b^2 e^2 n^2 \log[d (e + f x^2)^m] + 48 a b e^2 \log[c x^n] \log[d (e + f x^2)^m] + \\
& \quad 12 b^2 e^2 n \log[c x^n] \log[d (e + f x^2)^m] + 24 b^2 e^2 \log[c x^n]^2 \log[d (e + f x^2)^m] - \\
& \quad 12 b f^2 m n x^4 (4 a + b n + 4 b \log[c x^n]) \text{PolyLog}[2, - \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& \quad 12 b f^2 m n x^4 (4 a + b n + 4 b \log[c x^n]) \text{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& \quad \left. 48 b^2 f^2 m n^2 x^4 \text{PolyLog}[3, - \frac{i \sqrt{f} x}{\sqrt{e}}] + 48 b^2 f^2 m n^2 x^4 \text{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] \right)
\end{aligned}$$

**Problem 108: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int x (a + b \log[c x^n])^3 \log[d (e + f x^2)^m] dx$$

Optimal (type 4, 514 leaves, 26 steps):

$$\begin{aligned}
& \frac{3}{2} b^3 m n^3 x^2 - \frac{9}{4} b^2 m n^2 x^2 (a + b \log[c x^n]) + \frac{3}{2} b m n x^2 (a + b \log[c x^n])^2 - \\
& \frac{1}{2} m x^2 (a + b \log[c x^n])^3 - \frac{3 b^3 e m n^3 \log[e + f x^2]}{8 f} - \frac{3}{8} b^3 n^3 x^2 \log[d (e + f x^2)^m] + \\
& \frac{3}{4} b^2 n^2 x^2 (a + b \log[c x^n]) \log[d (e + f x^2)^m] - \frac{3}{4} b n x^2 (a + b \log[c x^n])^2 \log[d (e + f x^2)^m] + \\
& \frac{1}{2} x^2 (a + b \log[c x^n])^3 \log[d (e + f x^2)^m] + \frac{3 b^2 e m n^2 (a + b \log[c x^n]) \log[1 + \frac{f x^2}{e}]}{4 f} - \\
& \frac{3 b e m n (a + b \log[c x^n])^2 \log[1 + \frac{f x^2}{e}]}{4 f} + \frac{e m (a + b \log[c x^n])^3 \log[1 + \frac{f x^2}{e}]}{2 f} + \\
& \frac{3 b^3 e m n^3 \text{PolyLog}[2, -\frac{f x^2}{e}]}{8 f} - \frac{3 b^2 e m n^2 (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{f x^2}{e}]}{4 f} + \\
& \frac{3 b e m n (a + b \log[c x^n])^2 \text{PolyLog}[2, -\frac{f x^2}{e}]}{4 f} + \frac{3 b^3 e m n^3 \text{PolyLog}[3, -\frac{f x^2}{e}]}{8 f} - \\
& \frac{3 b^2 e m n^2 (a + b \log[c x^n]) \text{PolyLog}[3, -\frac{f x^2}{e}]}{4 f} + \frac{3 b^3 e m n^3 \text{PolyLog}[4, -\frac{f x^2}{e}]}{8 f}
\end{aligned}$$

Result (type 4, 1911 leaves):

$$\begin{aligned}
& \frac{1}{8 f} \left( -4 a^3 f m x^2 + 12 a^2 b f m n x^2 - 18 a b^2 f m n^2 x^2 + 12 b^3 f m n^3 x^2 - 12 a^2 b f m x^2 \log[c x^n] + \right. \\
& 24 a b^2 f m n x^2 \log[c x^n] - 18 b^3 f m n^2 x^2 \log[c x^n] - 12 a b^2 f m x^2 \log[c x^n]^2 + \\
& 12 b^3 f m n x^2 \log[c x^n]^2 - 4 b^3 f m x^2 \log[c x^n]^3 + 12 a^2 b e m n \log[x] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 12 a b^2 e m n^2 \log[x] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + 6 b^3 e m n^3 \log[x] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 12 a b^2 e m n^2 \log[x]^2 \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + 6 b^3 e m n^3 \log[x]^2 \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 4 b^3 e m n^3 \log[x]^3 \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + 24 a b^2 e m n \log[x] \log[c x^n] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 12 b^3 e m n^2 \log[x] \log[c x^n] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] - 12 b^3 e m n^2 \log[x]^2 \log[c x^n] \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 12 b^3 e m n \log[x] \log[c x^n]^2 \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + 12 a^2 b e m n \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 12 a b^2 e m n^2 \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + 6 b^3 e m n^3 \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 12 a b^2 e m n^2 \log[x]^2 \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + 6 b^3 e m n^3 \log[x]^2 \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}]
\end{aligned}$$

$$\begin{aligned}
& 4 b^3 e m n^3 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 24 a b^2 e m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 12 b^3 e m n^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 12 b^3 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 12 b^3 e m n \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 4 a^3 e m \operatorname{Log}[e + f x^2] - 6 a^2 b e m n \operatorname{Log}[e + f x^2] + \\
& 6 a b^2 e m n^2 \operatorname{Log}[e + f x^2] - 3 b^3 e m n^3 \operatorname{Log}[e + f x^2] - 12 a^2 b e m n \operatorname{Log}[x] \operatorname{Log}[e + f x^2] + \\
& 12 a b^2 e m n^2 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] - 6 b^3 e m n^3 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] + \\
& 12 a b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[e + f x^2] - 6 b^3 e m n^3 \operatorname{Log}[x]^2 \operatorname{Log}[e + f x^2] - \\
& 4 b^3 e m n^3 \operatorname{Log}[x]^3 \operatorname{Log}[e + f x^2] + 12 a^2 b e m \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] - \\
& 12 a b^2 e m n \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + 6 b^3 e m n^2 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] - \\
& 24 a b^2 e m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + 12 b^3 e m n^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + \\
& 12 b^3 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + 12 a b^2 e m \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x^2] - \\
& 6 b^3 e m n \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x^2] - 12 b^3 e m n \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x^2] + \\
& 4 b^3 e m \operatorname{Log}[c x^n]^3 \operatorname{Log}[e + f x^2] + 4 a^3 f x^2 \operatorname{Log}[d (e + f x^2)^m] - 6 a^2 b f n x^2 \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 a b^2 f n^2 x^2 \operatorname{Log}[d (e + f x^2)^m] - 3 b^3 f n^3 x^2 \operatorname{Log}[d (e + f x^2)^m] + \\
& 12 a^2 b f x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - 12 a b^2 f n x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 b^3 f n^2 x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + 12 a b^2 f x^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] - \\
& 6 b^3 f n x^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] + 4 b^3 f x^2 \operatorname{Log}[c x^n]^3 \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 b e m n \left(2 a^2 - 2 a b n + b^2 n^2 - 2 b (-2 a + b n) \operatorname{Log}[c x^n] + 2 b^2 \operatorname{Log}[c x^n]^2\right) \operatorname{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 6 b e m n \left(2 a^2 - 2 a b n + b^2 n^2 - 2 b (-2 a + b n) \operatorname{Log}[c x^n] + 2 b^2 \operatorname{Log}[c x^n]^2\right) \operatorname{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 24 a b^2 e m n^2 \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 12 b^3 e m n^3 \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 24 b^3 e m n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 24 a b^2 e m n^2 \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 12 b^3 e m n^3 \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] - 24 b^3 e m n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 24 b^3 e m n^3 \operatorname{PolyLog}[4, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 24 b^3 e m n^3 \operatorname{PolyLog}[4, \frac{i \sqrt{f} x}{\sqrt{e}}]
\end{aligned}$$

**Problem 109:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m]}{x} dx$$

Optimal (type 4, 181 leaves, 6 steps):

$$\frac{(a + b \operatorname{Log}[c x^n])^4 \operatorname{Log}[d (e + f x^2)^m]}{4 b n} - \frac{m (a + b \operatorname{Log}[c x^n])^4 \operatorname{Log}\left[1 + \frac{f x^2}{e}\right]}{4 b n} -$$

$$\frac{\frac{1}{2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}\left[2, -\frac{f x^2}{e}\right] + \frac{3}{4} b m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[3, -\frac{f x^2}{e}\right] -}{2}$$

$$\frac{3}{4} b^2 m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[4, -\frac{f x^2}{e}\right] + \frac{3}{8} b^3 m n^3 \operatorname{PolyLog}\left[5, -\frac{f x^2}{e}\right]$$

Result (type 4, 1348 leaves) :

$$-a^3 m \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \frac{3}{2} a^2 b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] -$$

$$a b^2 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \frac{1}{4} b^3 m n^3 \operatorname{Log}[x]^4 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] -$$

$$3 a^2 b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 3 a b^2 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] -$$

$$b^3 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 3 a b^2 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] +$$

$$\frac{3}{2} b^3 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - b^3 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] -$$

$$a^3 m \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \frac{3}{2} a^2 b m n \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] -$$

$$a b^2 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \frac{1}{4} b^3 m n^3 \operatorname{Log}[x]^4 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] -$$

$$3 a^2 b m \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 3 a b^2 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] -$$

$$b^3 m n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 3 a b^2 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] +$$

$$\frac{3}{2} b^3 m n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - b^3 m \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] +$$

$$a^3 \operatorname{Log}[x] \operatorname{Log}\left[d (e + f x^2)^m\right] - \frac{3}{2} a^2 b n \operatorname{Log}[x]^2 \operatorname{Log}\left[d (e + f x^2)^m\right] +$$

$$a b^2 n^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[d (e + f x^2)^m\right] - \frac{1}{4} b^3 n^3 \operatorname{Log}[x]^4 \operatorname{Log}\left[d (e + f x^2)^m\right] +$$

$$3 a^2 b \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[d (e + f x^2)^m\right] - 3 a b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[d (e + f x^2)^m\right] +$$

$$b^3 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}\left[d (e + f x^2)^m\right] + 3 a b^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[d (e + f x^2)^m\right] -$$

$$\frac{3}{2} b^3 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[d (e + f x^2)^m\right] + b^3 \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 \operatorname{Log}\left[d (e + f x^2)^m\right] -$$

$$m (a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right] - m (a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right] +$$

$$\begin{aligned}
& 3 a^2 b m n \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 6 a b^2 m n \log[c x^n] \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 3 b^3 m n \log[c x^n]^2 \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 3 a^2 b m n \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 6 a b^2 m n \log[c x^n] \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + 3 b^3 m n \log[c x^n]^2 \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 6 a b^2 m n^2 \operatorname{PolyLog}[4, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 6 b^3 m n^2 \log[c x^n] \operatorname{PolyLog}[4, -\frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 6 a b^2 m n^2 \operatorname{PolyLog}[4, \frac{i \sqrt{f} x}{\sqrt{e}}] - 6 b^3 m n^2 \log[c x^n] \operatorname{PolyLog}[4, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 6 b^3 m n^3 \operatorname{PolyLog}[5, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 6 b^3 m n^3 \operatorname{PolyLog}[5, \frac{i \sqrt{f} x}{\sqrt{e}}]
\end{aligned}$$

**Problem 110:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \log[c x^n])^3 \log[d (e + f x^2)^m]}{x^3} dx$$

Optimal (type 4, 451 leaves, 15 steps):

$$\begin{aligned}
& \frac{3 b^3 f m n^3 \log[x]}{4 e} - \frac{3 b^2 f m n^2 \log[1 + \frac{e}{f x^2}] (a + b \log[c x^n])}{4 e} - \\
& \frac{3 b f m n \log[1 + \frac{e}{f x^2}] (a + b \log[c x^n])^2}{4 e} - \frac{f m \log[1 + \frac{e}{f x^2}] (a + b \log[c x^n])^3}{2 e} - \\
& \frac{3 b^3 f m n^3 \log[e + f x^2]}{8 e} - \frac{3 b^3 n^3 \log[d (e + f x^2)^m]}{8 x^2} - \frac{3 b^2 n^2 (a + b \log[c x^n]) \log[d (e + f x^2)^m]}{4 x^2} - \\
& \frac{3 b n (a + b \log[c x^n])^2 \log[d (e + f x^2)^m]}{4 x^2} - \frac{(a + b \log[c x^n])^3 \log[d (e + f x^2)^m]}{2 x^2} + \\
& \frac{3 b^3 f m n^3 \operatorname{PolyLog}[2, -\frac{e}{f x^2}]}{8 e} + \frac{3 b^2 f m n^2 (a + b \log[c x^n]) \operatorname{PolyLog}[2, -\frac{e}{f x^2}]}{4 e} + \\
& \frac{3 b f m n (a + b \log[c x^n])^2 \operatorname{PolyLog}[2, -\frac{e}{f x^2}]}{4 e} + \frac{3 b^3 f m n^3 \operatorname{PolyLog}[3, -\frac{e}{f x^2}]}{8 e} + \\
& \frac{3 b^2 f m n^2 (a + b \log[c x^n]) \operatorname{PolyLog}[3, -\frac{e}{f x^2}]}{4 e} + \frac{3 b^3 f m n^3 \operatorname{PolyLog}[4, -\frac{e}{f x^2}]}{8 e}
\end{aligned}$$

Result (type 4, 2248 leaves):

$$\begin{aligned}
& -\frac{1}{8 e x^2} \left( -8 a^3 f m x^2 \log[x] - 12 a^2 b f m n x^2 \log[x] - 12 a b^2 f m n^2 x^2 \log[x] - 6 b^3 f m n^3 x^2 \log[x] + \right. \\
& 12 a^2 b f m n x^2 \log[x]^2 + 12 a b^2 f m n^2 x^2 \log[x]^2 + 6 b^3 f m n^3 x^2 \log[x]^2 - 8 a b^2 f m n^2 x^2 \log[x]^3 - \\
& 4 b^3 f m n^3 x^2 \log[x]^3 + 2 b^3 f m n^3 x^2 \log[x]^4 - 24 a^2 b f m x^2 \log[x] \log[c x^n] -
\end{aligned}$$

$$\begin{aligned}
& 24 a b^2 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] - 12 b^3 f m n^2 x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] + \\
& 24 a b^2 f m n x^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + 12 b^3 f m n^2 x^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] - \\
& 8 b^3 f m n^2 x^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] - 24 a b^2 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 - \\
& 12 b^3 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + 12 b^3 f m n x^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 - \\
& 8 b^3 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 + 12 a^2 b f m n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 12 a b^2 f m n^2 x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 b^3 f m n^3 x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 12 a b^2 f m n^2 x^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 6 b^3 f m n^3 x^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 4 b^3 f m n^3 x^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 24 a b^2 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 12 b^3 f m n^2 x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 12 b^3 f m n^2 x^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 12 b^3 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 12 a^2 b f m n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 12 a b^2 f m n^2 x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 b^3 f m n^3 x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 12 a b^2 f m n^2 x^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 6 b^3 f m n^3 x^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 4 b^3 f m n^3 x^2 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 24 a b^2 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 12 b^3 f m n^2 x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 12 b^3 f m n^2 x^2 \operatorname{Log}[x]^2 \\
& \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 12 b^3 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 4 a^3 f m x^2 \operatorname{Log}[e + f x^2] + 6 a^2 b f m n x^2 \operatorname{Log}[e + f x^2] + 6 a b^2 f m n^2 x^2 \operatorname{Log}[e + f x^2] + \\
& 3 b^3 f m n^3 x^2 \operatorname{Log}[e + f x^2] - 12 a^2 b f m n x^2 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] - \\
& 12 a b^2 f m n^2 x^2 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] - 6 b^3 f m n^3 x^2 \operatorname{Log}[x] \operatorname{Log}[e + f x^2] + \\
& 12 a b^2 f m n^2 x^2 \operatorname{Log}[x]^2 \operatorname{Log}[e + f x^2] + 6 b^3 f m n^3 x^2 \operatorname{Log}[x]^2 \operatorname{Log}[e + f x^2] - \\
& 4 b^3 f m n^3 x^2 \operatorname{Log}[x]^3 \operatorname{Log}[e + f x^2] + 12 a^2 b f m n x^2 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + \\
& 12 a b^2 f m n x^2 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + 6 b^3 f m n^2 x^2 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] - \\
& 24 a b^2 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] - 12 b^3 f m n^2 x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + \\
& 12 b^3 f m n^2 x^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}[e + f x^2] + 12 a b^2 f m n x^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x^2] + \\
& 6 b^3 f m n x^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x^2] - 12 b^3 f m n x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}[e + f x^2] + \\
& 4 b^3 f m n x^2 \operatorname{Log}[c x^n]^3 \operatorname{Log}[e + f x^2] + 4 a^3 e \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 a^2 b e n \operatorname{Log}[d (e + f x^2)^m] + 6 a b^2 e n^2 \operatorname{Log}[d (e + f x^2)^m] + 3 b^3 e n^3 \operatorname{Log}[d (e + f x^2)^m] +
\end{aligned}$$

$$\begin{aligned}
& 12 a^2 b e \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + 12 a b^2 e n \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 b^3 e n^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + 12 a b^2 e \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 b^3 e n \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] + 4 b^3 e \operatorname{Log}[c x^n]^3 \operatorname{Log}[d (e + f x^2)^m] + 6 b f m n x^2 \\
& \left(2 a^2 + 2 a b n + b^2 n^2 + 2 b (2 a + b n) \operatorname{Log}[c x^n] + 2 b^2 \operatorname{Log}[c x^n]^2\right) \operatorname{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 6 b f \\
& m n x^2 \left(2 a^2 + 2 a b n + b^2 n^2 + 2 b (2 a + b n) \operatorname{Log}[c x^n] + 2 b^2 \operatorname{Log}[c x^n]^2\right) \operatorname{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 24 a b^2 f m n^2 x^2 \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 12 b^3 f m n^3 x^2 \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 24 b^3 f m n^2 x^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 24 a b^2 f m n^2 x^2 \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 12 b^3 f m n^3 x^2 \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] - 24 b^3 f m n^2 x^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 24 b^3 f m n^3 x^2 \operatorname{PolyLog}[4, -\frac{i \sqrt{f} x}{\sqrt{e}}] + 24 b^3 f m n^3 x^2 \operatorname{PolyLog}[4, \frac{i \sqrt{f} x}{\sqrt{e}}]
\end{aligned}$$

**Problem 111: Result more than twice size of optimal antiderivative.**

$$\int x^2 (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m] dx$$

Optimal (type 4, 1092 leaves, 49 steps):

$$\begin{aligned}
& \frac{52 a b^2 e m n^2 x}{9 f} - \frac{160 b^3 e m n^3 x}{27 f} + \frac{16}{81} b^3 m n^3 x^3 + \frac{4 b^3 e^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{27 f^{3/2}} + \frac{52 b^3 e m n^2 x \operatorname{Log}[c x^n]}{9 f} - \\
& \frac{4}{9} b^2 m n^2 x^3 (a + b \operatorname{Log}[c x^n]) - \frac{4 b^2 e^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] (a + b \operatorname{Log}[c x^n])}{9 f^{3/2}} - \\
& \frac{8 b e m n x (a + b \operatorname{Log}[c x^n])^2}{3 f} + \frac{4}{9} b m n x^3 (a + b \operatorname{Log}[c x^n])^2 + \frac{2 e m x (a + b \operatorname{Log}[c x^n])^3}{3 f} - \\
& \frac{2}{9} m x^3 (a + b \operatorname{Log}[c x^n])^3 + \frac{b (-e)^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 f^{3/2}} - \\
& \frac{(-e)^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 f^{3/2}} - \frac{b (-e)^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 f^{3/2}} + \\
& \frac{(-e)^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 f^{3/2}} - \frac{2}{27} b^3 n^3 x^3 \operatorname{Log}[d (e + f x^2)^m] + \\
& \frac{2}{9} b^2 n^2 x^3 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m] - \frac{1}{3} b n x^3 (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m] + \\
& \frac{1}{3} x^3 (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m] - \frac{2 b^2 (-e)^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{3 f^{3/2}} + \\
& \frac{b (-e)^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{f^{3/2}} + \\
& \frac{2 b^2 (-e)^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, \frac{\sqrt{f} x}{\sqrt{-e}}]}{3 f^{3/2}} - \\
& \frac{b (-e)^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, \frac{\sqrt{f} x}{\sqrt{-e}}]}{f^{3/2}} + \frac{2 \pm b^3 e^{3/2} m n^3 \operatorname{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}]}{9 f^{3/2}} - \\
& \frac{2 \pm b^3 e^{3/2} m n^3 \operatorname{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}]}{9 f^{3/2}} + \frac{2 b^3 (-e)^{3/2} m n^3 \operatorname{PolyLog}[3, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{3 f^{3/2}} - \\
& \frac{2 b^2 (-e)^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{f^{3/2}} - \\
& \frac{2 b^3 (-e)^{3/2} m n^3 \operatorname{PolyLog}[3, \frac{\sqrt{f} x}{\sqrt{-e}}]}{3 f^{3/2}} + \frac{2 b^2 (-e)^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, \frac{\sqrt{f} x}{\sqrt{-e}}]}{f^{3/2}} + \\
& \frac{2 b^3 (-e)^{3/2} m n^3 \operatorname{PolyLog}[4, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{f^{3/2}} - \frac{2 b^3 (-e)^{3/2} m n^3 \operatorname{PolyLog}[4, \frac{\sqrt{f} x}{\sqrt{-e}}]}{f^{3/2}}
\end{aligned}$$

Result (type 4, 2544 leaves) :

$$\frac{1}{81 f^{3/2}} \left( 54 a^3 e \sqrt{f} m x - 216 a^2 b e \sqrt{f} m n x + 468 a b^2 e \sqrt{f} m n^2 x - 480 b^3 e \sqrt{f} m n^3 x - 18 a^3 f^{3/2} m x^3 + \right.$$

$$\begin{aligned}
& 36 a^2 b f^{3/2} m n x^3 - 36 a b^2 f^{3/2} m n^2 x^3 + 16 b^3 f^{3/2} m n^3 x^3 - 54 a^3 e^{3/2} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + \\
& 54 a^2 b e^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - 36 a b^2 e^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + 12 b^3 e^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + \\
& 162 a^2 b e^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] - 108 a b^2 e^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] + \\
& 36 b^3 e^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] - 162 a b^2 e^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 + \\
& 54 b^3 e^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 + 54 b^3 e^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^3 + \\
& 162 a^2 b e \sqrt{f} m x \operatorname{Log}[c x^n] - 432 a b^2 e \sqrt{f} m n x \operatorname{Log}[c x^n] + 468 b^3 e \sqrt{f} m n^2 x \operatorname{Log}[c x^n] - \\
& 54 a^2 b f^{3/2} m x^3 \operatorname{Log}[c x^n] + 72 a b^2 f^{3/2} m n x^3 \operatorname{Log}[c x^n] - 36 b^3 f^{3/2} m n^2 x^3 \operatorname{Log}[c x^n] - \\
& 162 a^2 b e^{3/2} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] + 108 a b^2 e^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] - \\
& 36 b^3 e^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] + 324 a b^2 e^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
& 108 b^3 e^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
& 162 b^3 e^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + 162 a b^2 e \sqrt{f} m x \operatorname{Log}[c x^n]^2 - \\
& 216 b^3 e \sqrt{f} m n x \operatorname{Log}[c x^n]^2 - 54 a b^2 f^{3/2} m x^3 \operatorname{Log}[c x^n]^2 + 36 b^3 f^{3/2} m n x^3 \operatorname{Log}[c x^n]^2 - \\
& 162 a b^2 e^{3/2} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 + 54 b^3 e^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 + \\
& 162 b^3 e^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + 54 b^3 e \sqrt{f} m x \operatorname{Log}[c x^n]^3 - \\
& 18 b^3 f^{3/2} m x^3 \operatorname{Log}[c x^n]^3 - 54 b^3 e^{3/2} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^3 - \\
& 81 i a^2 b e^{3/2} m n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 54 i a b^2 e^{3/2} m n^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 18 i b^3 e^{3/2} m n^3 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 81 i a b^2 e^{3/2} m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 27 i b^3 e^{3/2} m n^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 27 i b^3 e^{3/2} m n^3 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 162 i a b^2 e^{3/2} m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 54 i b^3 e^{3/2} m n^2 \operatorname{Log}[x] \\
& \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 81 i b^3 e^{3/2} m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] -
\end{aligned}$$

$$\begin{aligned}
& 81 \pm b^3 e^{3/2} m n \log[x] \log[c x^n]^2 \log[1 - \frac{i \sqrt{f} x}{\sqrt{e}}] + 81 \pm a^2 b e^{3/2} m n \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 54 \pm a b^2 e^{3/2} m n^2 \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + 18 \pm b^3 e^{3/2} m n^3 \log[x] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 81 \pm a b^2 e^{3/2} m n^2 \log[x]^2 \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + 27 \pm b^3 e^{3/2} m n^3 \log[x]^2 \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 27 \pm b^3 e^{3/2} m n^3 \log[x]^3 \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + 162 \pm a b^2 e^{3/2} m n \log[x] \log[c x^n] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 54 \pm b^3 e^{3/2} m n^2 \log[x] \log[c x^n] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 81 \pm b^3 e^{3/2} m n^2 \log[x]^2 \log[c x^n] \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 81 \pm b^3 e^{3/2} m n \log[x] \log[c x^n]^2 \log[1 + \frac{i \sqrt{f} x}{\sqrt{e}}] + 27 a^3 f^{3/2} x^3 \log[d (e + f x^2)^m] - \\
& 27 a^2 b f^{3/2} n x^3 \log[d (e + f x^2)^m] + 18 a b^2 f^{3/2} n^2 x^3 \log[d (e + f x^2)^m] - \\
& 6 b^3 f^{3/2} n^3 x^3 \log[d (e + f x^2)^m] + 81 a^2 b f^{3/2} x^3 \log[c x^n] \log[d (e + f x^2)^m] - \\
& 54 a b^2 f^{3/2} n x^3 \log[c x^n] \log[d (e + f x^2)^m] + 18 b^3 f^{3/2} n^2 x^3 \log[c x^n] \log[d (e + f x^2)^m] + \\
& 81 a b^2 f^{3/2} x^3 \log[c x^n]^2 \log[d (e + f x^2)^m] - 27 b^3 f^{3/2} n x^3 \log[c x^n]^2 \log[d (e + f x^2)^m] + \\
& 27 b^3 f^{3/2} x^3 \log[c x^n]^3 \log[d (e + f x^2)^m] + 9 \pm b e^{3/2} m n \\
& \left( 9 a^2 - 6 a b n + 2 b^2 n^2 - 6 b (-3 a + b n) \log[c x^n] + 9 b^2 \log[c x^n]^2 \right) \text{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}] - \\
& 9 \pm b e^{3/2} m n \left( 9 a^2 - 6 a b n + 2 b^2 n^2 - 6 b (-3 a + b n) \log[c x^n] + 9 b^2 \log[c x^n]^2 \right) \\
& \text{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}] - 162 \pm a b^2 e^{3/2} m n^2 \text{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 54 \pm b^3 e^{3/2} m n^3 \text{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 162 \pm b^3 e^{3/2} m n^2 \log[c x^n] \text{PolyLog}[3, -\frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 162 \pm a b^2 e^{3/2} m n^2 \text{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] - 54 \pm b^3 e^{3/2} m n^3 \text{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 162 \pm b^3 e^{3/2} m n^2 \log[c x^n] \text{PolyLog}[3, \frac{i \sqrt{f} x}{\sqrt{e}}] + \\
& 162 \pm b^3 e^{3/2} m n^3 \text{PolyLog}[4, -\frac{i \sqrt{f} x}{\sqrt{e}}] - 162 \pm b^3 e^{3/2} m n^3 \text{PolyLog}[4, \frac{i \sqrt{f} x}{\sqrt{e}}]
\end{aligned}$$

**Problem 112: Result more than twice size of optimal antiderivative.**

$$\int (a + b \log[c x^n])^3 \log[d (e + f x^2)^m] dx$$

Optimal (type 4, 977 leaves, 42 steps):

$$\begin{aligned}
& -24 a b^2 m n^2 x + 36 b^3 m n^3 x - 12 b^2 m n^2 (a - b n) x + \frac{12 b^2 \sqrt{e} m n^2 (a - b n) \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{\sqrt{f}} - \\
& 36 b^3 m n^2 x \operatorname{Log}[c x^n] + \frac{12 b^3 \sqrt{e} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]}{\sqrt{f}} + 12 b m n x (a + b \operatorname{Log}[c x^n])^2 - \\
& 2 m x (a + b \operatorname{Log}[c x^n])^3 + \frac{3 b \sqrt{-e} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{f}} - \\
& \frac{\sqrt{-e} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{f}} - \frac{3 b \sqrt{-e} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{f}} + \\
& \frac{\sqrt{-e} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{f}} + 6 a b^2 n^2 x \operatorname{Log}[d (e + f x^2)^m] - 6 b^3 n^3 x \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 b^3 n^2 x \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - 3 b n x (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m] + \\
& x (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m] - \frac{6 b^2 \sqrt{-e} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} + \\
& \frac{3 b \sqrt{-e} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} + \\
& \frac{6 b^2 \sqrt{-e} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, \frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} - \\
& \frac{3 b \sqrt{-e} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, \frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} - \frac{6 i b^3 \sqrt{e} m n^3 \operatorname{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}]}{\sqrt{f}} + \\
& \frac{6 i b^3 \sqrt{e} m n^3 \operatorname{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}]}{\sqrt{f}} + \frac{6 b^3 \sqrt{-e} m n^3 \operatorname{PolyLog}[3, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} - \\
& \frac{6 b^2 \sqrt{-e} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} - \\
& \frac{6 b^3 \sqrt{-e} m n^3 \operatorname{PolyLog}[3, \frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} + \frac{6 b^2 \sqrt{-e} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, \frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} + \\
& \frac{6 b^3 \sqrt{-e} m n^3 \operatorname{PolyLog}[4, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}} - \frac{6 b^3 \sqrt{-e} m n^3 \operatorname{PolyLog}[4, \frac{\sqrt{f} x}{\sqrt{-e}}]}{\sqrt{f}}
\end{aligned}$$

Result (type 4, 2302 leaves):

$$\frac{1}{\sqrt{f}} \left( -2 a^3 \sqrt{f} m x + 12 a^2 b \sqrt{f} m n x - 36 a b^2 \sqrt{f} m n^2 x + 48 b^3 \sqrt{f} m n^3 x + 2 a^3 \sqrt{e} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - \right)$$

$$\begin{aligned}
& 6 a^2 b \sqrt{e} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + 12 a b^2 \sqrt{e} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - 12 b^3 \sqrt{e} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - \\
& 6 a^2 b \sqrt{e} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] + 12 a b^2 \sqrt{e} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] - \\
& 12 b^3 \sqrt{e} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] + 6 a b^2 \sqrt{e} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 - \\
& 6 b^3 \sqrt{e} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 - 2 b^3 \sqrt{e} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^3 + \\
& 6 a^2 b \sqrt{f} m x \operatorname{Log}[c x^n] + 24 a b^2 \sqrt{f} m n x \operatorname{Log}[c x^n] - 36 b^3 \sqrt{f} m n^2 x \operatorname{Log}[c x^n] + \\
& 6 a^2 b \sqrt{e} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] - 12 a b^2 \sqrt{e} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] + \\
& 12 b^3 \sqrt{e} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] - 12 a b^2 \sqrt{e} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] + \\
& 12 b^3 \sqrt{e} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] + 6 b^3 \sqrt{e} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] - \\
& 6 a b^2 \sqrt{f} m x \operatorname{Log}[c x^n]^2 + 12 b^3 \sqrt{f} m n x \operatorname{Log}[c x^n]^2 + \\
& 6 a b^2 \sqrt{e} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 - 6 b^3 \sqrt{e} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 - \\
& 6 b^3 \sqrt{e} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 - 2 b^3 \sqrt{f} m x \operatorname{Log}[c x^n]^3 + \\
& 2 b^3 \sqrt{e} m \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^3 + 3 \pm a^2 b \sqrt{e} m n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 6 \pm a b^2 \sqrt{e} m n^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 \pm b^3 \sqrt{e} m n^3 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 3 \pm a b^2 \sqrt{e} m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 3 \pm b^3 \sqrt{e} m n^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& \pm b^3 \sqrt{e} m n^3 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 \pm a b^2 \sqrt{e} m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 6 \pm b^3 \sqrt{e} m n^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 3 \pm b^3 \sqrt{e} m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 3 \pm b^3 \sqrt{e} m n \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 3 \pm a^2 b \sqrt{e} m n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 6 \pm a b^2 \sqrt{e} m n^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 6 \pm b^3 \sqrt{e} m n^3 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] +
\end{aligned}$$

$$\begin{aligned}
& 3 \pm a b^2 \sqrt{e} m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - 3 \pm b^3 \sqrt{e} m n^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - \\
& \pm b^3 \sqrt{e} m n^3 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - 6 \pm a b^2 \sqrt{e} m n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 6 \pm b^3 \sqrt{e} m n^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + 3 \pm b^3 \sqrt{e} m n^2 \operatorname{Log}[x]^2 \\
& \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - 3 \pm b^3 \sqrt{e} m n \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& a^3 \sqrt{f} x \operatorname{Log}[d (e + f x^2)^m] - 3 a^2 b \sqrt{f} n x \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 a b^2 \sqrt{f} n^2 x \operatorname{Log}[d (e + f x^2)^m] - 6 b^3 \sqrt{f} n^3 x \operatorname{Log}[d (e + f x^2)^m] + \\
& 3 a^2 b \sqrt{f} x \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - 6 a b^2 \sqrt{f} n x \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + \\
& 6 b^3 \sqrt{f} n^2 x \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] + 3 a b^2 \sqrt{f} x \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] - \\
& 3 b^3 \sqrt{f} n x \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] + b^3 \sqrt{f} x \operatorname{Log}[c x^n]^3 \operatorname{Log}[d (e + f x^2)^m] - \\
& 3 \pm b \sqrt{e} m n \left( a^2 - 2 a b n + 2 b^2 n^2 + 2 b (a - b n) \operatorname{Log}[c x^n] + b^2 \operatorname{Log}[c x^n]^2 \right) \operatorname{PolyLog}[2, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] + \\
& 3 \pm b \sqrt{e} m n \left( a^2 - 2 a b n + 2 b^2 n^2 + 2 b (a - b n) \operatorname{Log}[c x^n] + b^2 \operatorname{Log}[c x^n]^2 \right) \operatorname{PolyLog}[2, \frac{\pm \sqrt{f} x}{\sqrt{e}}] + \\
& 6 \pm a b^2 \sqrt{e} m n^2 \operatorname{PolyLog}[3, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] - 6 \pm b^3 \sqrt{e} m n^3 \operatorname{PolyLog}[3, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] + \\
& 6 \pm b^3 \sqrt{e} m n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] - 6 \pm a b^2 \sqrt{e} m n^2 \operatorname{PolyLog}[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}] + \\
& 6 \pm b^3 \sqrt{e} m n^3 \operatorname{PolyLog}[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}] - 6 \pm b^3 \sqrt{e} m n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}] - \\
& 6 \pm b^3 \sqrt{e} m n^3 \operatorname{PolyLog}[4, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] + 6 \pm b^3 \sqrt{e} m n^3 \operatorname{PolyLog}[4, \frac{\pm \sqrt{f} x}{\sqrt{e}}]
\end{aligned}$$

**Problem 113: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m]}{x^2} dx$$

Optimal (type 4, 879 leaves, 26 steps):

$$\begin{aligned}
& \frac{12 b^3 \sqrt{f} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{\sqrt{e}} + \frac{12 b^2 \sqrt{f} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] (a + b \operatorname{Log}[c x^n])}{\sqrt{e}} + \\
& \frac{3 b \sqrt{f} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} + \frac{\sqrt{f} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \\
& \frac{3 b \sqrt{f} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \frac{\sqrt{f} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \\
& \frac{6 b^3 n^3 \operatorname{Log}\left[d (e + f x^2)^m\right]}{x} - \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[d (e + f x^2)^m\right]}{x} - \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[d (e + f x^2)^m\right]}{x} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[d (e + f x^2)^m\right]}{x} - \\
& \frac{6 b^2 \sqrt{f} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \\
& \frac{3 b \sqrt{f} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} + \\
& \frac{6 b^2 \sqrt{f} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} + \frac{3 b \sqrt{f} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \\
& \frac{6 i b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right]}{\sqrt{e}} + \frac{6 i b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right]}{\sqrt{e}} + \\
& \frac{6 b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} + \frac{6 b^2 \sqrt{f} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \\
& \frac{6 b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \frac{6 b^2 \sqrt{f} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} - \\
& \frac{6 b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[4, -\frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}} + \frac{6 b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[4, \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{\sqrt{-e}}
\end{aligned}$$

Result (type 4, 2166 leaves) :

$$\begin{aligned}
& \frac{1}{\sqrt{e} x} \\
& \left( 2 a^3 \sqrt{f} m x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + 6 a^2 b \sqrt{f} m n x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + 12 a b^2 \sqrt{f} m n^2 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + \right. \\
& \left. 12 b^3 \sqrt{f} m n^3 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - 6 a^2 b \sqrt{f} m n x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] - \right)
\end{aligned}$$

$$\begin{aligned}
& 12 a b^2 \sqrt{f} m n^2 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] - 12 b^3 \sqrt{f} m n^3 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] + \\
& 6 a b^2 \sqrt{f} m n^2 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 + 6 b^3 \sqrt{f} m n^3 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 - \\
& 2 b^3 \sqrt{f} m n^3 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^3 + 6 a^2 b \sqrt{f} m x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] + \\
& 12 a b^2 \sqrt{f} m n x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] + 12 b^3 \sqrt{f} m n^2 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] - \\
& 12 a b^2 \sqrt{f} m n x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
& 12 b^3 \sqrt{f} m n^2 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] + \\
& 6 b^3 \sqrt{f} m n^2 x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + 6 a b^2 \sqrt{f} m x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 + \\
& 6 b^3 \sqrt{f} m n x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 - 6 b^3 \sqrt{f} m n x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + \\
& 2 b^3 \sqrt{f} m x \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^3 + 3 i a^2 b \sqrt{f} m n x \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 6 i a b^2 \sqrt{f} m n^2 x \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 i b^3 \sqrt{f} m n^3 x \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 3 i a b^2 \sqrt{f} m n^2 x \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 3 i b^3 \sqrt{f} m n^3 x \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& i b^3 \sqrt{f} m n^3 x \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 6 i a b^2 \sqrt{f} m n x \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 6 i b^3 \sqrt{f} m n^2 x \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 3 i b^3 \sqrt{f} m n^2 x \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 3 i b^3 \sqrt{f} m n x \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 3 i a^2 b \sqrt{f} m n x \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 6 i a b^2 \sqrt{f} m n^2 x \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 6 i b^3 \sqrt{f} m n^3 x \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 3 i a b^2 \sqrt{f} m n^2 x \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 3 i b^3 \sqrt{f} m n^3 x \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& i b^3 \sqrt{f} m n^3 x \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 6 i a b^2 \sqrt{f} m n x \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{i \sqrt{f} x}{\sqrt{e}}\right] -
\end{aligned}$$

$$\begin{aligned}
& 6 \pm b^3 \sqrt{f} m n^2 x \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 3 \pm b^3 \sqrt{f} m n^2 x \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - \\
& 3 \pm b^3 \sqrt{f} m n x \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - a^3 \sqrt{e} \operatorname{Log}\left[d (e + f x^2)^m\right] - \\
& 3 a^2 b \sqrt{e} n \operatorname{Log}\left[d (e + f x^2)^m\right] - 6 a b^2 \sqrt{e} n^2 \operatorname{Log}\left[d (e + f x^2)^m\right] - 6 b^3 \sqrt{e} n^3 \operatorname{Log}\left[d (e + f x^2)^m\right] - \\
& 3 a^2 b \sqrt{e} \operatorname{Log}[c x^n] \operatorname{Log}\left[d (e + f x^2)^m\right] - 6 a b^2 \sqrt{e} n \operatorname{Log}[c x^n] \operatorname{Log}\left[d (e + f x^2)^m\right] - \\
& 6 b^3 \sqrt{e} n^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[d (e + f x^2)^m\right] - 3 a b^2 \sqrt{e} \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[d (e + f x^2)^m\right] - \\
& 3 b^3 \sqrt{e} n \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[d (e + f x^2)^m\right] - b^3 \sqrt{e} \operatorname{Log}[c x^n]^3 \operatorname{Log}\left[d (e + f x^2)^m\right] - 3 \pm b \sqrt{f} m \\
& n x \left( a^2 + 2 a b n + 2 b^2 n^2 + 2 b (a + b n) \operatorname{Log}[c x^n] + b^2 \operatorname{Log}[c x^n]^2 \right) \operatorname{PolyLog}\left[2, - \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 3 \pm b \sqrt{f} m n x \left( a^2 + 2 a b n + 2 b^2 n^2 + 2 b (a + b n) \operatorname{Log}[c x^n] + b^2 \operatorname{Log}[c x^n]^2 \right) \operatorname{PolyLog}\left[2, \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 6 \pm a b^2 \sqrt{f} m n^2 \operatorname{PolyLog}\left[3, - \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + 6 \pm b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[3, - \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 6 \pm b^3 \sqrt{f} m n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}\left[3, - \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - 6 \pm a b^2 \sqrt{f} m n^2 \operatorname{PolyLog}\left[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - \\
& 6 \pm b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - 6 \pm b^3 \sqrt{f} m n^2 \operatorname{Log}[c x^n] \operatorname{PolyLog}\left[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - \\
& 6 \pm b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[4, - \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + 6 \pm b^3 \sqrt{f} m n^3 \operatorname{PolyLog}\left[4, \frac{\pm \sqrt{f} x}{\sqrt{e}}\right]
\end{aligned}$$

**Problem 114: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[d (e + f x^2)^m\right]}{x^4} dx$$

Optimal (type 4, 1007 leaves, 36 steps):

$$\begin{aligned}
& - \frac{160 b^3 f m n^3}{27 e x} - \frac{4 b^3 f^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{27 e^{3/2}} - \frac{52 b^2 f m n^2 (a + b \operatorname{Log}[c x^n])}{9 e x} - \\
& \frac{4 b^2 f^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] (a + b \operatorname{Log}[c x^n])}{9 e^{3/2}} - \frac{8 b f m n (a + b \operatorname{Log}[c x^n])^2}{3 e x} - \\
& \frac{2 f m (a + b \operatorname{Log}[c x^n])^3}{3 e x} + \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} - \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} - \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} - \frac{2 b^3 n^3 \operatorname{Log}[d (e + f x^2)^m]}{27 x^3} - \\
& \frac{2 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{9 x^3} - \frac{b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{3 (-e)^{3/2}} - \\
& \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{(-e)^{3/2}} + \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, \frac{\sqrt{f} x}{\sqrt{-e}}]}{3 (-e)^{3/2}} + \\
& \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, \frac{\sqrt{f} x}{\sqrt{-e}}]}{(-e)^{3/2}} + \frac{2 i b^3 f^{3/2} m n^3 \operatorname{PolyLog}[2, -\frac{i \sqrt{f} x}{\sqrt{e}}]}{9 e^{3/2}} - \\
& \frac{2 i b^3 f^{3/2} m n^3 \operatorname{PolyLog}[2, \frac{i \sqrt{f} x}{\sqrt{e}}]}{9 e^{3/2}} + \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}[3, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{3 (-e)^{3/2}} + \\
& \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{(-e)^{3/2}} - \\
& \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}[3, \frac{\sqrt{f} x}{\sqrt{-e}}]}{3 (-e)^{3/2}} - \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, \frac{\sqrt{f} x}{\sqrt{-e}}]}{(-e)^{3/2}} - \\
& \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}[4, -\frac{\sqrt{f} x}{\sqrt{-e}}]}{(-e)^{3/2}} + \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}[4, \frac{\sqrt{f} x}{\sqrt{-e}}]}{(-e)^{3/2}}
\end{aligned}$$

Result (type 4, 2488 leaves):

$$\begin{aligned}
& \frac{1}{27 e^{3/2} x^3} \left( -18 a^3 \sqrt{e} f m x^2 - 72 a^2 b \sqrt{e} f m n x^2 - 156 a b^2 \sqrt{e} f m n^2 x^2 - \right. \\
& 160 b^3 \sqrt{e} f m n^3 x^2 - 18 a^3 f^{3/2} m x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - 18 a^2 b f^{3/2} m n x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - 
\end{aligned}$$

$$\begin{aligned}
& 12 a b^2 f^{3/2} m n^2 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] - 4 b^3 f^{3/2} m n^3 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] + \\
& 54 a^2 b f^{3/2} m n x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] + 36 a b^2 f^{3/2} m n^2 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] + \\
& 12 b^3 f^{3/2} m n^3 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] - 54 a b^2 f^{3/2} m n^2 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 - \\
& 18 b^3 f^{3/2} m n^3 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 + 18 b^3 f^{3/2} m n^3 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^3 - \\
& 54 a^2 b \sqrt{e} f m x^2 \operatorname{Log}[c x^n] - 144 a b^2 \sqrt{e} f m n x^2 \operatorname{Log}[c x^n] - 156 b^3 \sqrt{e} f m n^2 x^2 \operatorname{Log}[c x^n] - \\
& 54 a^2 b f^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] - 36 a b^2 f^{3/2} m n x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] - \\
& 12 b^3 f^{3/2} m n^2 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n] + 108 a b^2 f^{3/2} m n x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] + \\
& 36 b^3 f^{3/2} m n^2 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
& 54 b^3 f^{3/2} m n^2 x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] - 54 a b^2 \sqrt{e} f m x^2 \operatorname{Log}[c x^n]^2 - \\
& 72 b^3 \sqrt{e} f m n x^2 \operatorname{Log}[c x^n]^2 - 54 a b^2 f^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 - \\
& 18 b^3 f^{3/2} m n x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^2 + 54 b^3 f^{3/2} m n x^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 - \\
& 18 b^3 \sqrt{e} f m x^2 \operatorname{Log}[c x^n]^3 - 18 b^3 f^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] \operatorname{Log}[c x^n]^3 - \\
& 27 \pm a^2 b f^{3/2} m n x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 18 \pm a b^2 f^{3/2} m n^2 x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 6 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + 27 \pm a b^2 f^{3/2} m n^2 x^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 9 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - 9 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 54 \pm a b^2 f^{3/2} m n x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] - \\
& 18 \pm b^3 f^{3/2} m n^2 x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] + \\
& 27 \pm b^3 f^{3/2} m n^2 x^3 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 - \frac{i \sqrt{f} x}{\sqrt{e}}\right] -
\end{aligned}$$

$$\begin{aligned}
& 27 \pm b^3 f^{3/2} m n x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 - \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 27 \pm a^2 b f^{3/2} m n x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + 18 \pm a b^2 f^{3/2} m n^2 x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 6 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - 27 \pm a b^2 f^{3/2} m n^2 x^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - \\
& 9 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + 9 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 54 \pm a b^2 f^{3/2} m n x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 18 \pm b^3 f^{3/2} m n^2 x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - \\
& 27 \pm b^3 f^{3/2} m n^2 x^3 \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] + \\
& 27 \pm b^3 f^{3/2} m n x^3 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}\left[1 + \frac{\pm \sqrt{f} x}{\sqrt{e}}\right] - 9 a^3 e^{3/2} \operatorname{Log}[d (e + f x^2)^m] - \\
& 9 a^2 b e^{3/2} n \operatorname{Log}[d (e + f x^2)^m] - 6 a b^2 e^{3/2} n^2 \operatorname{Log}[d (e + f x^2)^m] - 2 b^3 e^{3/2} n^3 \operatorname{Log}[d (e + f x^2)^m] - \\
& 27 a^2 b e^{3/2} \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - 18 a b^2 e^{3/2} n \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - \\
& 6 b^3 e^{3/2} n^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^2)^m] - 27 a b^2 e^{3/2} \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] - \\
& 9 b^3 e^{3/2} n \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^2)^m] - 9 b^3 e^{3/2} \operatorname{Log}[c x^n]^3 \operatorname{Log}[d (e + f x^2)^m] + 3 \pm b f^{3/2} m n x^3 \\
& \left(9 a^2 + 6 a b n + 2 b^2 n^2 + 6 b (3 a + b n) \operatorname{Log}[c x^n] + 9 b^2 \operatorname{Log}[c x^n]^2\right) \operatorname{PolyLog}[2, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] - \\
& 3 \pm b f^{3/2} m n x^3 \left(9 a^2 + 6 a b n + 2 b^2 n^2 + 6 b (3 a + b n) \operatorname{Log}[c x^n] + 9 b^2 \operatorname{Log}[c x^n]^2\right) \\
& \operatorname{PolyLog}[2, \frac{\pm \sqrt{f} x}{\sqrt{e}}] - 54 \pm a b^2 f^{3/2} m n^2 x^3 \operatorname{PolyLog}[3, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] - \\
& 18 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{PolyLog}[3, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] - 54 \pm b^3 f^{3/2} m n^2 x^3 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] + \\
& 54 \pm a b^2 f^{3/2} m n^2 x^3 \operatorname{PolyLog}[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}] + 18 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{PolyLog}[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}] + \\
& 54 \pm b^3 f^{3/2} m n^2 x^3 \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, \frac{\pm \sqrt{f} x}{\sqrt{e}}] + \\
& 54 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{PolyLog}[4, -\frac{\pm \sqrt{f} x}{\sqrt{e}}] - 54 \pm b^3 f^{3/2} m n^3 x^3 \operatorname{PolyLog}[4, \frac{\pm \sqrt{f} x}{\sqrt{e}}]
\end{aligned}$$

**Problem 125:** Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] (\text{a} + \text{b} \text{Log}[\text{c} x^n])^2}{x} dx$$

Optimal (type 4, 145 leaves, 5 steps):

$$\begin{aligned} & \frac{\text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] (\text{a} + \text{b} \text{Log}[\text{c} x^n])^3}{3 \text{b} n} - \\ & \frac{\text{Log}[1 + \frac{\text{f} \sqrt{x}}{\text{e}}] (\text{a} + \text{b} \text{Log}[\text{c} x^n])^3}{3 \text{b} n} - 2 (\text{a} + \text{b} \text{Log}[\text{c} x^n])^2 \text{PolyLog}[2, -\frac{\text{f} \sqrt{x}}{\text{e}}] + \\ & 8 \text{b} n (\text{a} + \text{b} \text{Log}[\text{c} x^n]) \text{PolyLog}[3, -\frac{\text{f} \sqrt{x}}{\text{e}}] - 16 \text{b}^2 n^2 \text{PolyLog}[4, -\frac{\text{f} \sqrt{x}}{\text{e}}] \end{aligned}$$

Result (type 4, 368 leaves):

$$\begin{aligned} & \text{a}^2 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x] - \text{a}^2 \text{Log}[1 + \frac{\text{f} \sqrt{x}}{\text{e}}] \text{Log}[x] - \text{a} \text{b} n \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^2 + \\ & \text{a} \text{b} n \text{Log}[1 + \frac{\text{f} \sqrt{x}}{\text{e}}] \text{Log}[x]^2 + \frac{1}{3} \text{b}^2 n^2 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^3 - \\ & \frac{1}{3} \text{b}^2 n^2 \text{Log}[1 + \frac{\text{f} \sqrt{x}}{\text{e}}] \text{Log}[x]^3 + 2 \text{a} \text{b} \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x] \text{Log}[\text{c} x^n] - \\ & 2 \text{a} \text{b} \text{Log}[1 + \frac{\text{f} \sqrt{x}}{\text{e}}] \text{Log}[x] \text{Log}[\text{c} x^n] - \text{b}^2 n \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^2 \text{Log}[\text{c} x^n] + \\ & \text{b}^2 n \text{Log}[1 + \frac{\text{f} \sqrt{x}}{\text{e}}] \text{Log}[x]^2 \text{Log}[\text{c} x^n] + \text{b}^2 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x] \text{Log}[\text{c} x^n]^2 - \\ & \text{b}^2 \text{Log}[1 + \frac{\text{f} \sqrt{x}}{\text{e}}] \text{Log}[x] \text{Log}[\text{c} x^n]^2 - 2 (\text{a} + \text{b} \text{Log}[\text{c} x^n])^2 \text{PolyLog}[2, -\frac{\text{f} \sqrt{x}}{\text{e}}] + \\ & 8 \text{b} n (\text{a} + \text{b} \text{Log}[\text{c} x^n]) \text{PolyLog}[3, -\frac{\text{f} \sqrt{x}}{\text{e}}] - 16 \text{b}^2 n^2 \text{PolyLog}[4, -\frac{\text{f} \sqrt{x}}{\text{e}}] \end{aligned}$$

### Problem 128: Result more than twice size of optimal antiderivative.

$$\int x \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] (\text{a} + \text{b} \text{Log}[\text{c} x^n])^3 dx$$

Optimal (type 4, 907 leaves, 36 steps):

$$\begin{aligned}
& -\frac{255 b^3 e^3 n^3 \sqrt{x}}{8 f^3} - \frac{9 a b^2 e^2 n^2 x}{4 f^2} + \frac{45 b^3 e^2 n^3 x}{16 f^2} - \frac{175 b^3 e n^3 x^{3/2}}{216 f} + \frac{3}{8} b^3 n^3 x^2 + \\
& \frac{3 b^3 e^4 n^3 \text{Log}[e + f \sqrt{x}]}{8 f^4} - \frac{3}{8} b^3 n^3 x^2 \text{Log}[d(e + f \sqrt{x})] + \frac{3 b^3 e^4 n^3 \text{Log}[e + f \sqrt{x}] \text{Log}[-\frac{f \sqrt{x}}{e}]}{2 f^4} - \\
& \frac{9 b^3 e^2 n^2 x \text{Log}[c x^n]}{4 f^2} + \frac{63 b^2 e^3 n^2 \sqrt{x} (a + b \text{Log}[c x^n])}{4 f^3} - \frac{3 b^2 e^2 n^2 x (a + b \text{Log}[c x^n])}{8 f^2} + \\
& \frac{37 b^2 e n^2 x^{3/2} (a + b \text{Log}[c x^n])}{36 f} - \frac{9}{16} b^2 n^2 x^2 (a + b \text{Log}[c x^n]) - \\
& \frac{3 b^2 e^4 n^2 \text{Log}[e + f \sqrt{x}] (a + b \text{Log}[c x^n])}{4 f^4} + \frac{3}{4} b^2 n^2 x^2 \text{Log}[d(e + f \sqrt{x})] (a + b \text{Log}[c x^n]) - \\
& \frac{15 b e^3 n \sqrt{x} (a + b \text{Log}[c x^n])^2}{4 f^3} + \frac{9 b e^2 n x (a + b \text{Log}[c x^n])^2}{8 f^2} - \frac{7 b e n x^{3/2} (a + b \text{Log}[c x^n])^2}{12 f} + \\
& \frac{3}{8} b n x^2 (a + b \text{Log}[c x^n])^2 - \frac{3}{4} b n x^2 \text{Log}[d(e + f \sqrt{x})] (a + b \text{Log}[c x^n])^2 + \\
& \frac{3 b e^4 n \text{Log}[1 + \frac{f \sqrt{x}}{e}] (a + b \text{Log}[c x^n])^2}{4 f^4} + \frac{e^3 \sqrt{x} (a + b \text{Log}[c x^n])^3}{2 f^3} - \\
& \frac{e^2 x (a + b \text{Log}[c x^n])^3}{4 f^2} + \frac{e x^{3/2} (a + b \text{Log}[c x^n])^3}{6 f} - \frac{1}{8} x^2 (a + b \text{Log}[c x^n])^3 + \\
& \frac{1}{2} x^2 \text{Log}[d(e + f \sqrt{x})] (a + b \text{Log}[c x^n])^3 - \frac{e^4 \text{Log}[1 + \frac{f \sqrt{x}}{e}] (a + b \text{Log}[c x^n])^3}{2 f^4} + \\
& \frac{3 b^3 e^4 n^3 \text{PolyLog}[2, 1 + \frac{f \sqrt{x}}{e}]}{2 f^4} + \frac{3 b^2 e^4 n^2 (a + b \text{Log}[c x^n]) \text{PolyLog}[2, -\frac{f \sqrt{x}}{e}]}{f^4} - \\
& \frac{3 b e^4 n (a + b \text{Log}[c x^n])^2 \text{PolyLog}[2, -\frac{f \sqrt{x}}{e}]}{f^4} - \frac{6 b^3 e^4 n^3 \text{PolyLog}[3, -\frac{f \sqrt{x}}{e}]}{f^4} + \\
& \frac{12 b^2 e^4 n^2 (a + b \text{Log}[c x^n]) \text{PolyLog}[3, -\frac{f \sqrt{x}}{e}]}{f^4} - \frac{24 b^3 e^4 n^3 \text{PolyLog}[4, -\frac{f \sqrt{x}}{e}]}{f^4}
\end{aligned}$$

Result (type 4, 1968 leaves) :

$$\begin{aligned}
& \frac{1}{432 f^4} \\
& \left( 216 a^3 e^3 f \sqrt{x} - 1620 a^2 b e^3 f n \sqrt{x} + 6804 a b^2 e^3 f n^2 \sqrt{x} - 13770 b^3 e^3 f n^3 \sqrt{x} - 108 a^3 e^2 f^2 x + \right. \\
& 486 a^2 b e^2 f^2 n x - 1134 a b^2 e^2 f^2 n^2 x + 1215 b^3 e^2 f^2 n^3 x + 72 a^3 e f^3 x^{3/2} - 252 a^2 b e f^3 n x^{3/2} + \\
& 444 a b^2 e f^3 n^2 x^{3/2} - 350 b^3 e f^3 n^3 x^{3/2} - 54 a^3 f^4 x^2 + 162 a^2 b f^4 n x^2 - 243 a b^2 f^4 n^2 x^2 + \\
& 162 b^3 f^4 n^3 x^2 - 216 a^3 e^4 \text{Log}[e + f \sqrt{x}] + 324 a^2 b e^4 n \text{Log}[e + f \sqrt{x}] - \\
& 324 a b^2 e^4 n^2 \text{Log}[e + f \sqrt{x}] + 162 b^3 e^4 n^3 \text{Log}[e + f \sqrt{x}] + 216 a^3 f^4 x^2 \text{Log}[d(e + f \sqrt{x})] - \\
& 324 a^2 b f^4 n x^2 \text{Log}[d(e + f \sqrt{x})] + 324 a b^2 f^4 n^2 x^2 \text{Log}[d(e + f \sqrt{x})] - \\
& 162 b^3 f^4 n^3 x^2 \text{Log}[d(e + f \sqrt{x})] + 648 a^2 b e^4 n \text{Log}[e + f \sqrt{x}] \text{Log}[x] - \\
& 648 a b^2 e^4 n^2 \text{Log}[e + f \sqrt{x}] \text{Log}[x] + 324 b^3 e^4 n^3 \text{Log}[e + f \sqrt{x}] \text{Log}[x] -
\end{aligned}$$

$$\begin{aligned}
& 648 a^2 b e^4 n \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x] + 648 a b^2 e^4 n^2 \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x] - \\
& 324 b^3 e^4 n^3 \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x] - 648 a b^2 e^4 n^2 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[x]^2 + \\
& 324 b^3 e^4 n^3 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[x]^2 + 648 a b^2 e^4 n^2 \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x]^2 - \\
& 324 b^3 e^4 n^3 \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x]^2 + 216 b^3 e^4 n^3 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[x]^3 - \\
& 216 b^3 e^4 n^3 \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x]^3 + 648 a^2 b e^3 f \sqrt{x} \operatorname{Log}[c x^n] - \\
& 3240 a b^2 e^3 f n \sqrt{x} \operatorname{Log}[c x^n] + 6804 b^3 e^3 f n^2 \sqrt{x} \operatorname{Log}[c x^n] - 324 a^2 b e^2 f^2 x \operatorname{Log}[c x^n] + \\
& 972 a b^2 e^2 f^2 n x \operatorname{Log}[c x^n] - 1134 b^3 e^2 f^2 n^2 x \operatorname{Log}[c x^n] + 216 a^2 b e f^3 x^{3/2} \operatorname{Log}[c x^n] - \\
& 504 a b^2 e f^3 n x^{3/2} \operatorname{Log}[c x^n] + 444 b^3 e f^3 n^2 x^{3/2} \operatorname{Log}[c x^n] - 162 a^2 b f^4 x^2 \operatorname{Log}[c x^n] + \\
& 324 a b^2 f^4 n x^2 \operatorname{Log}[c x^n] - 243 b^3 f^4 n^2 x^2 \operatorname{Log}[c x^n] - 648 a^2 b e^4 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[c x^n] + \\
& 648 a b^2 e^4 n \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[c x^n] - 324 b^3 e^4 n^2 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[c x^n] + \\
& 648 a^2 b f^4 x^2 \operatorname{Log}[d(e + f \sqrt{x})] \operatorname{Log}[c x^n] - 648 a b^2 f^4 n x^2 \operatorname{Log}[d(e + f \sqrt{x})] \operatorname{Log}[c x^n] + \\
& 324 b^3 f^4 n^2 x^2 \operatorname{Log}[d(e + f \sqrt{x})] \operatorname{Log}[c x^n] + 1296 a b^2 e^4 n \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
& 648 b^3 e^4 n^2 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[x] \operatorname{Log}[c x^n] - 1296 a b^2 e^4 n \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] + \\
& 648 b^3 e^4 n^2 \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n] - 648 b^3 e^4 n^2 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + \\
& 648 b^3 e^4 n^2 \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + 648 a b^2 e^3 f \sqrt{x} \operatorname{Log}[c x^n]^2 - \\
& 1620 b^3 e^3 f n \sqrt{x} \operatorname{Log}[c x^n]^2 - 324 a b^2 e^2 f^2 x \operatorname{Log}[c x^n]^2 + 486 b^3 e^2 f^2 n x \operatorname{Log}[c x^n]^2 + \\
& 216 a b^2 e f^3 x^{3/2} \operatorname{Log}[c x^n]^2 - 252 b^3 e f^3 n x^{3/2} \operatorname{Log}[c x^n]^2 - 162 a b^2 f^4 x^2 \operatorname{Log}[c x^n]^2 + \\
& 162 b^3 f^4 n x^2 \operatorname{Log}[c x^n]^2 - 648 a b^2 e^4 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[c x^n]^2 + \\
& 324 b^3 e^4 n \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[c x^n]^2 + 648 a b^2 f^4 x^2 \operatorname{Log}[d(e + f \sqrt{x})] \operatorname{Log}[c x^n]^2 - \\
& 324 b^3 f^4 n x^2 \operatorname{Log}[d(e + f \sqrt{x})] \operatorname{Log}[c x^n]^2 + 648 b^3 e^4 n \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 - \\
& 648 b^3 e^4 n \operatorname{Log}\left[1 + \frac{f \sqrt{x}}{e}\right] \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + 216 b^3 e^3 f \sqrt{x} \operatorname{Log}[c x^n]^3 - \\
& 108 b^3 e^2 f^2 x \operatorname{Log}[c x^n]^3 + 72 b^3 e f^3 x^{3/2} \operatorname{Log}[c x^n]^3 - 54 b^3 f^4 x^2 \operatorname{Log}[c x^n]^3 - \\
& 216 b^3 e^4 \operatorname{Log}[e + f \sqrt{x}] \operatorname{Log}[c x^n]^3 + 216 b^3 f^4 x^2 \operatorname{Log}[d(e + f \sqrt{x})] \operatorname{Log}[c x^n]^3 - \\
& 648 b e^4 n \left(2 a^2 - 2 a b n + b^2 n^2 - 2 b (-2 a + b n) \operatorname{Log}[c x^n] + 2 b^2 \operatorname{Log}[c x^n]^2\right) \operatorname{PolyLog}[2, -\frac{f \sqrt{x}}{e}] + \\
& 2592 b^2 e^4 n^2 (2 a - b n + 2 b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{f \sqrt{x}}{e}] - 10368 b^3 e^4 n^3 \operatorname{PolyLog}[4, -\frac{f \sqrt{x}}{e}]
\end{aligned}$$

**Problem 129: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Log}[d(e + f \sqrt{x})] (a + b \operatorname{Log}[c x^n])^3 dx$$

Optimal (type 4, 639 leaves, 30 steps):

$$\begin{aligned}
 & -\frac{90 b^3 e n^3 \sqrt{x}}{f} - 6 a b^2 n^2 x + 12 b^3 n^3 x + \frac{6 b^3 e^2 n^3 \log[e + f \sqrt{x}]}{f^2} - 6 b^3 n^3 x \log[d(e + f \sqrt{x})] + \\
 & \frac{12 b^3 e^2 n^3 \log[e + f \sqrt{x}] \log[-\frac{f \sqrt{x}}{e}]}{f^2} - 6 b^3 n^2 x \log[c x^n] + \frac{42 b^2 e n^2 \sqrt{x} (a + b \log[c x^n])}{f} - \\
 & 3 b^2 n^2 x (a + b \log[c x^n]) - \frac{6 b^2 e^2 n^2 \log[e + f \sqrt{x}] (a + b \log[c x^n])}{f^2} + \\
 & 6 b^2 n^2 x \log[d(e + f \sqrt{x})] (a + b \log[c x^n]) - \frac{9 b e n \sqrt{x} (a + b \log[c x^n])^2}{f} + \\
 & 3 b n x (a + b \log[c x^n])^2 - 3 b n x \log[d(e + f \sqrt{x})] (a + b \log[c x^n])^2 + \\
 & \frac{3 b e^2 n \log[1 + \frac{f \sqrt{x}}{e}] (a + b \log[c x^n])^2}{f^2} + \frac{e \sqrt{x} (a + b \log[c x^n])^3}{f} - \frac{1}{2} x (a + b \log[c x^n])^3 + \\
 & x \log[d(e + f \sqrt{x})] (a + b \log[c x^n])^3 - \frac{e^2 \log[1 + \frac{f \sqrt{x}}{e}] (a + b \log[c x^n])^3}{f^2} + \\
 & \frac{12 b^3 e^2 n^3 \text{PolyLog}[2, 1 + \frac{f \sqrt{x}}{e}]}{f^2} + \frac{12 b^2 e^2 n^2 (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{f \sqrt{x}}{e}]}{f^2} - \\
 & \frac{6 b e^2 n (a + b \log[c x^n])^2 \text{PolyLog}[2, -\frac{f \sqrt{x}}{e}]}{f^2} - \frac{24 b^3 e^2 n^3 \text{PolyLog}[3, -\frac{f \sqrt{x}}{e}]}{f^2} + \\
 & \frac{24 b^2 e^2 n^2 (a + b \log[c x^n]) \text{PolyLog}[3, -\frac{f \sqrt{x}}{e}]}{f^2} - \frac{48 b^3 e^2 n^3 \text{PolyLog}[4, -\frac{f \sqrt{x}}{e}]}{f^2}
 \end{aligned}$$

Result (type 4, 1513 leaves):

$$\begin{aligned}
 & \frac{1}{2} x (-a^3 + 3 a^2 b n - 6 a b^2 n^2 + 6 b^3 n^3 - 3 a^2 b (-n \log[x] + \log[c x^n])) + \\
 & 6 a b^2 n (-n \log[x] + \log[c x^n]) - 6 b^3 n^2 (-n \log[x] + \log[c x^n]) - \\
 & 3 a b^2 (-n \log[x] + \log[c x^n])^2 + 3 b^3 n (-n \log[x] + \log[c x^n])^2 - b^3 (-n \log[x] + \log[c x^n])^3 + \\
 & \frac{1}{f} e \sqrt{x} (a^3 - 3 a^2 b n + 6 a b^2 n^2 - 6 b^3 n^3 + 3 a^2 b (-n \log[x] + \log[c x^n])) - \\
 & 6 a b^2 n (-n \log[x] + \log[c x^n]) + 6 b^3 n^2 (-n \log[x] + \log[c x^n]) + \\
 & 3 a b^2 (-n \log[x] + \log[c x^n])^2 - 3 b^3 n (-n \log[x] + \log[c x^n])^2 + b^3 (-n \log[x] + \log[c x^n])^3 - \\
 & \frac{1}{f^2} e^2 \log[e + f \sqrt{x}] (a^3 - 3 a^2 b n + 6 a b^2 n^2 - 6 b^3 n^3 + 3 a^2 b (-n \log[x] + \log[c x^n])) - \\
 & 6 a b^2 n (-n \log[x] + \log[c x^n]) + 6 b^3 n^2 (-n \log[x] + \log[c x^n]) + \\
 & 3 a b^2 (-n \log[x] + \log[c x^n])^2 - 3 b^3 n (-n \log[x] + \log[c x^n])^2 + b^3 (-n \log[x] + \log[c x^n])^3 + \\
 & x \log[d(e + f \sqrt{x})] (a^3 - 3 a^2 b n + 6 a b^2 n^2 - 6 b^3 n^3 + 3 a^2 b n \log[x] - 6 a b^2 n^2 \log[x] + \\
 & 6 b^3 n^3 \log[x] + 3 a b^2 n^2 \log[x]^2 - 3 b^3 n^3 \log[x]^2 + b^3 n^3 \log[x]^3 + \\
 & 3 a^2 b (-n \log[x] + \log[c x^n]) - 6 a b^2 n (-n \log[x] + \log[c x^n]) + \\
 & 6 b^3 n^2 (-n \log[x] + \log[c x^n]) + 6 a b^2 n \log[x] (-n \log[x] + \log[c x^n]) - \\
 & 6 b^3 n^2 \log[x] (-n \log[x] + \log[c x^n]) + 3 b^3 n^2 \log[x]^2 (-n \log[x] + \log[c x^n]) + \\
 & 3 a b^2 (-n \log[x] + \log[c x^n])^2 - 3 b^3 n (-n \log[x] + \log[c x^n])^2 +
 \end{aligned}$$

$$\begin{aligned}
& 3 b^3 n \operatorname{Log}[x] (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 + b^3 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^3 \Big) - \\
& 3 b f n (a^2 - 2 a b n + 2 b^2 n^2 + 2 a b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) - \\
& 2 b^2 n (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) + b^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 \Big) \\
& \left( \left( -\frac{e \sqrt{x}}{f^2} + \frac{x}{2 f} + \frac{e^2 \operatorname{Log}[e + f \sqrt{x}]}{f^3} \right) (-2 \operatorname{Log}[\sqrt{x}] + \operatorname{Log}[x]) + 2 \left( -\frac{e \sqrt{x} (-1 + \operatorname{Log}[\sqrt{x}])}{f^2} + \right. \right. \\
& \left. \left. -\frac{\frac{x}{4} + \frac{1}{2} x \operatorname{Log}[\sqrt{x}]}{f} + \frac{e^2 (\operatorname{Log}[1 + \frac{f \sqrt{x}}{e}] \operatorname{Log}[\sqrt{x}] + \operatorname{PolyLog}[2, -\frac{f \sqrt{x}}{e}])}{f^3} \right) \right) + \\
& 3 b^2 f n^2 (-a + b n - b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) \left( \left( -\frac{e \sqrt{x}}{f^2} + \frac{x}{2 f} + \frac{e^2 \operatorname{Log}[e + f \sqrt{x}]}{f^3} \right) \right. \\
& (-2 \operatorname{Log}[\sqrt{x}] + \operatorname{Log}[x])^2 + 4 (-2 \operatorname{Log}[\sqrt{x}] + \operatorname{Log}[x]) \left( -\frac{e \sqrt{x} (-1 + \operatorname{Log}[\sqrt{x}])}{f^2} + \right. \\
& \left. \left. -\frac{\frac{x}{4} + \frac{1}{2} x \operatorname{Log}[\sqrt{x}]}{f} + \frac{e^2 (\operatorname{Log}[1 + \frac{f \sqrt{x}}{e}] \operatorname{Log}[\sqrt{x}] + \operatorname{PolyLog}[2, -\frac{f \sqrt{x}}{e}])}{f^3} \right) + \frac{1}{f^3} \right) \\
& 4 \left( -e f \sqrt{x} (2 - 2 \operatorname{Log}[\sqrt{x}] + \operatorname{Log}[\sqrt{x}]^2) + \frac{1}{4} f^2 x (1 - 2 \operatorname{Log}[\sqrt{x}] + 2 \operatorname{Log}[\sqrt{x}]^2) + e^2 \left( \operatorname{Log}[\right. \right. \\
& \left. \left. 1 + \frac{f \sqrt{x}}{e}] \operatorname{Log}[\sqrt{x}]^2 + 2 \operatorname{Log}[\sqrt{x}] \operatorname{PolyLog}[2, -\frac{f \sqrt{x}}{e}] - 2 \operatorname{PolyLog}[3, -\frac{f \sqrt{x}}{e}] \right) \right) - \\
& \frac{1}{2 (e + f \sqrt{x})} b^3 f n^3 \left( 1 + \frac{f \sqrt{x}}{e} \right) x^{3/2} \left( \frac{e (-2 e f \sqrt{x} + f^2 x + 2 e^2 \operatorname{Log}[1 + \frac{f \sqrt{x}}{e}]) \operatorname{Log}[x]^3}{f^3 x^{3/2}} - \right. \\
& \left. \frac{3 e \operatorname{Log}[x]^2 (-4 e f \sqrt{x} + f^2 x - 4 e^2 \operatorname{PolyLog}[2, -\frac{f \sqrt{x}}{e}])}{f^3 x^{3/2}} + \right. \\
& \left. \frac{6 e \operatorname{Log}[x] (-8 e f \sqrt{x} + f^2 x - 8 e^2 \operatorname{PolyLog}[3, -\frac{f \sqrt{x}}{e}])}{f^3 x^{3/2}} - \right. \\
& \left. \frac{6 e (-16 e f \sqrt{x} + f^2 x - 16 e^2 \operatorname{PolyLog}[4, -\frac{f \sqrt{x}}{e}])}{f^3 x^{3/2}} \right)
\end{aligned}$$

Problem 130: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] (a + b \text{Log}[c x^n])^3}{x} dx$$

Optimal (type 4, 178 leaves, 6 steps):

$$\begin{aligned} & \frac{\text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] (a + b \text{Log}[c x^n])^4}{4 b n} - \frac{\text{Log}[1 + \frac{f \sqrt{x}}{e}] (a + b \text{Log}[c x^n])^4}{4 b n} - \\ & 2 (a + b \text{Log}[c x^n])^3 \text{PolyLog}[2, -\frac{f \sqrt{x}}{e}] + 12 b n (a + b \text{Log}[c x^n])^2 \text{PolyLog}[3, -\frac{f \sqrt{x}}{e}] - \\ & 48 b^2 n^2 (a + b \text{Log}[c x^n]) \text{PolyLog}[4, -\frac{f \sqrt{x}}{e}] + 96 b^3 n^3 \text{PolyLog}[5, -\frac{f \sqrt{x}}{e}] \end{aligned}$$

Result (type 4, 662 leaves):

$$\begin{aligned} & \frac{1}{4} \left( 4 a^3 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x] - 4 a^3 \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x] - 6 a^2 b n \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^2 + \right. \\ & 6 a^2 b n \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x]^2 + 4 a b^2 n^2 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^3 - \\ & 4 a b^2 n^2 \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x]^3 - b^3 n^3 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^4 + \\ & b^3 n^3 \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x]^4 + 12 a^2 b \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x] \text{Log}[c x^n] - \\ & 12 a^2 b \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x] \text{Log}[c x^n] - 12 a b^2 n \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^2 \text{Log}[c x^n] + \\ & 12 a b^2 n \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x]^2 \text{Log}[c x^n] + 4 b^3 n^2 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^3 \text{Log}[c x^n] - \\ & 4 b^3 n^2 \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x]^3 \text{Log}[c x^n] + 12 a b^2 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x] \text{Log}[c x^n]^2 - \\ & 12 a b^2 \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x] \text{Log}[c x^n]^2 - 6 b^3 n \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x]^2 \text{Log}[c x^n]^2 + \\ & 6 b^3 n \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x]^2 \text{Log}[c x^n]^2 + 4 b^3 \text{Log}[\text{d}(\text{e} + \text{f} \sqrt{x})] \text{Log}[x] \text{Log}[c x^n]^3 - \\ & 4 b^3 \text{Log}[1 + \frac{f \sqrt{x}}{e}] \text{Log}[x] \text{Log}[c x^n]^3 - 8 (a + b \text{Log}[c x^n])^3 \text{PolyLog}[2, -\frac{f \sqrt{x}}{e}] + \\ & 48 b n (a + b \text{Log}[c x^n])^2 \text{PolyLog}[3, -\frac{f \sqrt{x}}{e}] - 192 a b^2 n^2 \text{PolyLog}[4, -\frac{f \sqrt{x}}{e}] - \\ & \left. 192 b^3 n^2 \text{Log}[c x^n] \text{PolyLog}[4, -\frac{f \sqrt{x}}{e}] + 384 b^3 n^3 \text{PolyLog}[5, -\frac{f \sqrt{x}}{e}] \right) \end{aligned}$$

Problem 139: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \text{Log}[c x^n])^3 \text{Log}[\text{d}(\text{e} + \text{f} x^m)^r]}{x} dx$$

Optimal (type 4, 185 leaves, 6 steps) :

$$\begin{aligned} & \frac{(a+b \log[c x^n])^4 \log[d (e+f x^m)^r]}{4 b n} - \frac{r (a+b \log[c x^n])^4 \log[1+\frac{f x^m}{e}]}{4 b n} - \\ & \frac{r (a+b \log[c x^n])^3 \text{PolyLog}[2, -\frac{f x^m}{e}]}{m} + \frac{3 b n r (a+b \log[c x^n])^2 \text{PolyLog}[3, -\frac{f x^m}{e}]}{m^2} - \\ & \frac{6 b^2 n^2 r (a+b \log[c x^n]) \text{PolyLog}[4, -\frac{f x^m}{e}]}{m^3} + \frac{6 b^3 n^3 r \text{PolyLog}[5, -\frac{f x^m}{e}]}{m^4} \end{aligned}$$

Result (type 4, 1395 leaves) :

$$\begin{aligned} & -\frac{1}{2} a^2 b m n r \log[x]^3 + \frac{3}{4} a b^2 m n^2 r \log[x]^4 - \frac{3}{10} b^3 m n^3 r \log[x]^5 - \\ & a b^2 m n r \log[x]^3 \log[c x^n] + \frac{3}{4} b^3 m n^2 r \log[x]^4 \log[c x^n] - \frac{1}{2} b^3 m n r \log[x]^3 \log[c x^n]^2 - \\ & \frac{3}{2} a^2 b n r \log[x]^2 \log[1+\frac{e x^{-m}}{f}] + 2 a b^2 n^2 r \log[x]^3 \log[1+\frac{e x^{-m}}{f}] - \\ & \frac{3}{4} b^3 n^3 r \log[x]^4 \log[1+\frac{e x^{-m}}{f}] - 3 a b^2 n r \log[x]^2 \log[c x^n] \log[1+\frac{e x^{-m}}{f}] + \\ & 2 b^3 n^2 r \log[x]^3 \log[c x^n] \log[1+\frac{e x^{-m}}{f}] - \frac{3}{2} b^3 n r \log[x]^2 \log[c x^n]^2 \log[1+\frac{e x^{-m}}{f}] - \\ & a^3 r \log[x] \log[e+f x^m] + 3 a^2 b n r \log[x]^2 \log[e+f x^m] - 3 a b^2 n^2 r \log[x]^3 \log[e+f x^m] + \\ & b^3 n^3 r \log[x]^4 \log[e+f x^m] + \frac{a^3 r \log[-\frac{f x^m}{e}] \log[e+f x^m]}{m} - \\ & \frac{3 a^2 b n r \log[x] \log[-\frac{f x^m}{e}] \log[e+f x^m]}{m} + \frac{3 a b^2 n^2 r \log[x]^2 \log[-\frac{f x^m}{e}] \log[e+f x^m]}{m} - \\ & \frac{b^3 n^3 r \log[x]^3 \log[-\frac{f x^m}{e}] \log[e+f x^m]}{m} - 3 a^2 b r \log[x] \log[c x^n] \log[e+f x^m] + \\ & 6 a b^2 n r \log[x]^2 \log[c x^n] \log[e+f x^m] - 3 b^3 n^2 r \log[x]^3 \log[c x^n] \log[e+f x^m] + \\ & \frac{3 a^2 b r \log[-\frac{f x^m}{e}] \log[c x^n] \log[e+f x^m]}{m} - \frac{6 a b^2 n r \log[x] \log[-\frac{f x^m}{e}] \log[c x^n] \log[e+f x^m]}{m} + \\ & \frac{3 b^3 n^2 r \log[x]^2 \log[-\frac{f x^m}{e}] \log[c x^n] \log[e+f x^m]}{m} - 3 a b^2 r \log[x] \log[c x^n]^2 \log[e+f x^m] + \\ & 3 b^3 n r \log[x]^2 \log[c x^n]^2 \log[e+f x^m] + \frac{3 a b^2 r \log[-\frac{f x^m}{e}] \log[c x^n]^2 \log[e+f x^m]}{m} - \\ & \frac{3 b^3 n r \log[x] \log[-\frac{f x^m}{e}] \log[c x^n]^2 \log[e+f x^m]}{m} - b^3 r \log[x] \log[c x^n]^3 \log[e+f x^m] + \\ & \frac{b^3 r \log[-\frac{f x^m}{e}] \log[c x^n]^3 \log[e+f x^m]}{m} + a^3 \log[x] \log[d (e+f x^m)^r] - \\ & \frac{3}{2} a^2 b n \log[x]^2 \log[d (e+f x^m)^r] + a b^2 n^2 \log[x]^3 \log[d (e+f x^m)^r] - \\ & \frac{1}{4} b^3 n^3 \log[x]^4 \log[d (e+f x^m)^r] + 3 a^2 b \log[x] \log[c x^n] \log[d (e+f x^m)^r] - \end{aligned}$$

$$\begin{aligned}
& 3 a b^2 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^m)^r] + b^3 n^2 \operatorname{Log}[x]^3 \operatorname{Log}[c x^n] \operatorname{Log}[d (e + f x^m)^r] + \\
& 3 a b^2 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^m)^r] - \frac{3}{2} b^3 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[d (e + f x^m)^r] + \\
& b^3 \operatorname{Log}[x] \operatorname{Log}[c x^n]^3 \operatorname{Log}[d (e + f x^m)^r] + \frac{1}{m} b n r \operatorname{Log}[x] \\
& \left( b^2 n^2 \operatorname{Log}[x]^2 - 3 b n \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n]) + 3 (a + b \operatorname{Log}[c x^n])^2 \right) \operatorname{PolyLog}[2, -\frac{e x^{-m}}{f}] + \\
& \frac{r (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}[2, 1 + \frac{f x^m}{e}]}{m} + \frac{3 a^2 b n r \operatorname{PolyLog}[3, -\frac{e x^{-m}}{f}]}{m^2} + \\
& \frac{6 a b^2 n r \operatorname{Log}[c x^n] \operatorname{PolyLog}[3, -\frac{e x^{-m}}{f}]}{m^2} + \frac{3 b^3 n r \operatorname{Log}[c x^n]^2 \operatorname{PolyLog}[3, -\frac{e x^{-m}}{f}]}{m^2} + \\
& \frac{6 a b^2 n^2 r \operatorname{PolyLog}[4, -\frac{e x^{-m}}{f}]}{m^3} + \frac{6 b^3 n^2 r \operatorname{Log}[c x^n] \operatorname{PolyLog}[4, -\frac{e x^{-m}}{f}]}{m^3} + \frac{6 b^3 n^3 r \operatorname{PolyLog}[5, -\frac{e x^{-m}}{f}]}{m^4}
\end{aligned}$$

**Problem 140: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^m)^r]}{x} dx$$

Optimal (type 4, 150 leaves, 5 steps) :

$$\begin{aligned}
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^m)^r]}{3 b n} - \\
& \frac{r (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + \frac{f x^m}{e}]}{3 b n} - \frac{r (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -\frac{f x^m}{e}]}{m} + \\
& \frac{2 b n r (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -\frac{f x^m}{e}]}{m^2} - \frac{2 b^2 n^2 r \operatorname{PolyLog}[4, -\frac{f x^m}{e}]}{m^3}
\end{aligned}$$

Result (type 4, 741 leaves) :

$$\begin{aligned}
& -\frac{1}{3} a b m n r \log[x]^3 + \frac{1}{4} b^2 m n^2 r \log[x]^4 - \frac{1}{3} b^2 m n r \log[x]^3 \log[c x^n] - \\
& a b n r \log[x]^2 \log\left[1 + \frac{e x^{-m}}{f}\right] + \frac{2}{3} b^2 n^2 r \log[x]^3 \log\left[1 + \frac{e x^{-m}}{f}\right] - \\
& b^2 n r \log[x]^2 \log[c x^n] \log\left[1 + \frac{e x^{-m}}{f}\right] - a^2 r \log[x] \log[e + f x^m] + \\
& 2 a b n r \log[x]^2 \log[e + f x^m] - b^2 n^2 r \log[x]^3 \log[e + f x^m] + \frac{a^2 r \log\left[-\frac{f x^m}{e}\right] \log[e + f x^m]}{m} - \\
& 2 a b n r \log[x] \log\left[-\frac{f x^m}{e}\right] \log[e + f x^m] + \frac{b^2 n^2 r \log[x]^2 \log\left[-\frac{f x^m}{e}\right] \log[e + f x^m]}{m} - \\
& 2 a b r \log[x] \log[c x^n] \log[e + f x^m] + 2 b^2 n r \log[x]^2 \log[c x^n] \log[e + f x^m] + \\
& 2 a b r \log\left[-\frac{f x^m}{e}\right] \log[c x^n] \log[e + f x^m] - \frac{2 b^2 n r \log[x] \log\left[-\frac{f x^m}{e}\right] \log[c x^n] \log[e + f x^m]}{m} - \\
& b^2 r \log[x] \log[c x^n]^2 \log[e + f x^m] + \frac{b^2 r \log\left[-\frac{f x^m}{e}\right] \log[c x^n]^2 \log[e + f x^m]}{m} + \\
& a^2 \log[x] \log[d (e + f x^m)^r] - a b n \log[x]^2 \log[d (e + f x^m)^r] + \\
& \frac{1}{3} b^2 n^2 \log[x]^3 \log[d (e + f x^m)^r] + 2 a b \log[x] \log[c x^n] \log[d (e + f x^m)^r] - \\
& b^2 n \log[x]^2 \log[c x^n] \log[d (e + f x^m)^r] + b^2 \log[x] \log[c x^n]^2 \log[d (e + f x^m)^r] + \\
& \frac{b n r \log[x] (-b n \log[x] + 2 (a + b \log[c x^n]))) \text{PolyLog}[2, -\frac{e x^{-m}}{f}]}{m} + \\
& \frac{r (a - b n \log[x] + b \log[c x^n])^2 \text{PolyLog}[2, 1 + \frac{f x^m}{e}]}{m} + \frac{2 a b n r \text{PolyLog}[3, -\frac{e x^{-m}}{f}]}{m^2} + \\
& \frac{2 b^2 n r \log[c x^n] \text{PolyLog}[3, -\frac{e x^{-m}}{f}]}{m^2} + \frac{2 b^2 n^2 r \text{PolyLog}[4, -\frac{e x^{-m}}{f}]}{m^3}
\end{aligned}$$

**Problem 141: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \log[c x^n]) \log[d (e + f x^m)^r]}{x} dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$\begin{aligned}
& \frac{(a + b \log[c x^n])^2 \log[d (e + f x^m)^r]}{2 b n} - \frac{r (a + b \log[c x^n])^2 \log[1 + \frac{f x^m}{e}]}{2 b n} - \\
& \frac{r (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{f x^m}{e}]}{m} + \frac{b n r \text{PolyLog}[3, -\frac{f x^m}{e}]}{m^2}
\end{aligned}$$

Result (type 4, 304 leaves):

$$\begin{aligned}
& -\frac{1}{6} b m n r \log[x]^3 - \frac{1}{2} b n r \log[x]^2 \log\left[1 + \frac{e x^{-m}}{f}\right] - a r \log[x] \log[e + f x^m] + \\
& b n r \log[x]^2 \log[e + f x^m] + \frac{a r \log\left[-\frac{f x^m}{e}\right] \log[e + f x^m]}{m} - \frac{b n r \log[x] \log\left[-\frac{f x^m}{e}\right] \log[e + f x^m]}{m} - \\
& b r \log[x] \log[c x^n] \log[e + f x^m] + \frac{b r \log\left[-\frac{f x^m}{e}\right] \log[c x^n] \log[e + f x^m]}{m} + \\
& a \log[x] \log[d (e + f x^m)^r] - \frac{1}{2} b n \log[x]^2 \log[d (e + f x^m)^r] + \\
& b \log[x] \log[c x^n] \log[d (e + f x^m)^r] + \frac{b n r \log[x] \text{PolyLog}[2, -\frac{e x^{-m}}{f}]}{m} + \\
& r (a - b n \log[x] + b \log[c x^n]) \text{PolyLog}[2, 1 + \frac{f x^m}{e}] + \frac{b n r \text{PolyLog}[3, -\frac{e x^{-m}}{f}]}{m^2}
\end{aligned}$$

**Problem 147: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \log[c x^n]) \log[d (e + f x^m)^k]}{x} dx$$

Optimal (type 4, 114 leaves, 4 steps) :

$$\begin{aligned}
& \frac{(a + b \log[c x^n])^2 \log[d (e + f x^m)^k]}{2 b n} - \frac{k (a + b \log[c x^n])^2 \log[1 + \frac{f x^m}{e}]}{2 b n} - \\
& \frac{k (a + b \log[c x^n]) \text{PolyLog}[2, -\frac{f x^m}{e}]}{m} + \frac{b k n \text{PolyLog}[3, -\frac{f x^m}{e}]}{m^2}
\end{aligned}$$

Result (type 4, 304 leaves) :

$$\begin{aligned}
& -\frac{1}{6} b k m n \log[x]^3 - \frac{1}{2} b k n \log[x]^2 \log\left[1 + \frac{e x^{-m}}{f}\right] - a k \log[x] \log[e + f x^m] + \\
& b k n \log[x]^2 \log[e + f x^m] + \frac{a k \log\left[-\frac{f x^m}{e}\right] \log[e + f x^m]}{m} - \frac{b k n \log[x] \log\left[-\frac{f x^m}{e}\right] \log[e + f x^m]}{m} - \\
& b k \log[x] \log[c x^n] \log[e + f x^m] + \frac{b k \log\left[-\frac{f x^m}{e}\right] \log[c x^n] \log[e + f x^m]}{m} + \\
& a \log[x] \log[d (e + f x^m)^k] - \frac{1}{2} b n \log[x]^2 \log[d (e + f x^m)^k] + \\
& b \log[x] \log[c x^n] \log[d (e + f x^m)^k] + \frac{b k n \log[x] \text{PolyLog}[2, -\frac{e x^{-m}}{f}]}{m} + \\
& k (a - b n \log[x] + b \log[c x^n]) \text{PolyLog}[2, 1 + \frac{f x^m}{e}] + \frac{b k n \text{PolyLog}[3, -\frac{e x^{-m}}{f}]}{m^2}
\end{aligned}$$

### Problem 166: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 (d + e \operatorname{Log}[f x^r])}{x} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{e r (a + b \operatorname{Log}[c x^n])^4}{12 b^2 n^2} + \frac{(a + b \operatorname{Log}[c x^n])^3 (d + e \operatorname{Log}[f x^r])}{3 b n}$$

Result (type 3, 129 leaves):

$$\begin{aligned} & \frac{1}{12} \operatorname{Log}[x] \left( -3 b^2 e n^2 r \operatorname{Log}[x]^3 + 12 (a + b \operatorname{Log}[c x^n])^2 (d + e \operatorname{Log}[f x^r]) + \right. \\ & \quad 4 b n \operatorname{Log}[x]^2 (b d n + 2 a e r + 2 b e r \operatorname{Log}[c x^n] + b e n \operatorname{Log}[f x^r]) - \\ & \quad \left. 6 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n]) (2 b d n + a e r + b e r \operatorname{Log}[c x^n] + 2 b e n \operatorname{Log}[f x^r]) \right) \end{aligned}$$

### Problem 199: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}[k, e x^q]}{x} dx$$

Optimal (type 4, 104 leaves, 4 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}[1+k, e x^q]}{q} - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2+k, e x^q]}{q^2} + \\ & \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3+k, e x^q]}{q^3} - \frac{6 b^3 n^3 \operatorname{PolyLog}[4+k, e x^q]}{q^4} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{PolyLog}[k, e x^q]}{x} dx$$

### Problem 200: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[k, e x^q]}{x} dx$$

Optimal (type 4, 72 leaves, 3 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[1+k, e x^q]}{q} - \\ & \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2+k, e x^q]}{q^2} + \frac{2 b^2 n^2 \operatorname{PolyLog}[3+k, e x^q]}{q^3} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[k, e x^q]}{x} dx$$

### Problem 201: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[k, e x^q]}{x} dx$$

Optimal (type 4, 40 leaves, 2 steps):

$$\frac{(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[1 + k, e x^q]}{q} - \frac{b n \operatorname{PolyLog}[2 + k, e x^q]}{q^2}$$

Result (type 8, 23 leaves):

$$\int \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[k, e x^q]}{x} dx$$

### Problem 205: Unable to integrate problem.

$$\int \frac{\operatorname{Log}[x] \operatorname{PolyLog}[n, a x]}{x} dx$$

Optimal (type 4, 20 leaves, 2 steps):

$$\operatorname{Log}[x] \operatorname{PolyLog}[1 + n, a x] - \operatorname{PolyLog}[2 + n, a x]$$

Result (type 8, 13 leaves):

$$\int \frac{\operatorname{Log}[x] \operatorname{PolyLog}[n, a x]}{x} dx$$

### Problem 206: Unable to integrate problem.

$$\int \frac{\operatorname{Log}[x]^2 \operatorname{PolyLog}[n, a x]}{x} dx$$

Optimal (type 4, 33 leaves, 3 steps):

$$\operatorname{Log}[x]^2 \operatorname{PolyLog}[1 + n, a x] - 2 \operatorname{Log}[x] \operatorname{PolyLog}[2 + n, a x] + 2 \operatorname{PolyLog}[3 + n, a x]$$

Result (type 8, 15 leaves):

$$\int \frac{\operatorname{Log}[x]^2 \operatorname{PolyLog}[n, a x]}{x} dx$$

### Problem 207: Unable to integrate problem.

$$\int \left( \frac{q \operatorname{PolyLog}[-1 + k, e x^q]}{b n x (a + b \operatorname{Log}[c x^n])} - \frac{\operatorname{PolyLog}[k, e x^q]}{x (a + b \operatorname{Log}[c x^n])^2} \right) dx$$

Optimal (type 4, 26 leaves, 2 steps):

$$\frac{\operatorname{PolyLog}[k, e x^q]}{b n (a + b \operatorname{Log}[c x^n])}$$

Result (type 8, 59 leaves):

$$\int \left( \frac{q \operatorname{PolyLog}[-1+k, e x^q]}{b n x (a + b \operatorname{Log}[c x^n])} - \frac{\operatorname{PolyLog}[k, e x^q]}{x (a + b \operatorname{Log}[c x^n])^2} \right) dx$$

Problem 214: Unable to integrate problem.

$$\int x^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x] dx$$

Optimal (type 4, 253 leaves, 15 steps):

$$\begin{aligned} & -\frac{2 b n x}{27 e^2} - \frac{b n x^2}{36 e} - \frac{4}{243} b n x^3 + \frac{x (a + b \operatorname{Log}[c x^n])}{27 e^2} + \frac{x^2 (a + b \operatorname{Log}[c x^n])}{54 e} + \\ & \frac{1}{81} x^3 (a + b \operatorname{Log}[c x^n]) - \frac{b n \operatorname{Log}[1 - e x]}{27 e^3} + \frac{1}{27} b n x^3 \operatorname{Log}[1 - e x] + \\ & \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 - e x]}{27 e^3} - \frac{1}{27} x^3 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 - e x] + \\ & \frac{b n \operatorname{PolyLog}[2, e x]}{27 e^3} + \frac{2}{27} b n x^3 \operatorname{PolyLog}[2, e x] - \frac{1}{9} x^3 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, e x] - \\ & \frac{1}{9} b n x^3 \operatorname{PolyLog}[3, e x] + \frac{1}{3} x^3 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x] \end{aligned}$$

Result (type 8, 21 leaves):

$$\int x^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x] dx$$

Problem 215: Unable to integrate problem.

$$\int x (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x] dx$$

Optimal (type 4, 221 leaves, 15 steps):

$$\begin{aligned} & -\frac{5 b n x}{16 e} - \frac{1}{8} b n x^2 + \frac{x (a + b \operatorname{Log}[c x^n])}{8 e} + \frac{1}{16} x^2 (a + b \operatorname{Log}[c x^n]) - \frac{3 b n \operatorname{Log}[1 - e x]}{16 e^2} + \\ & \frac{3}{16} b n x^2 \operatorname{Log}[1 - e x] + \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 - e x]}{8 e^2} - \frac{1}{8} x^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 - e x] + \\ & \frac{b n \operatorname{PolyLog}[2, e x]}{8 e^2} + \frac{1}{4} b n x^2 \operatorname{PolyLog}[2, e x] - \frac{1}{4} x^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, e x] - \\ & \frac{1}{4} b n x^2 \operatorname{PolyLog}[3, e x] + \frac{1}{2} x^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x] \end{aligned}$$

Result (type 8, 19 leaves):

$$\int x (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x] dx$$

### Problem 216: Unable to integrate problem.

$$\int (a + b \log[c x^n]) \operatorname{PolyLog}[3, e x] dx$$

Optimal (type 4, 131 leaves, 14 steps):

$$\begin{aligned} & -4 b n x + x (a + b \log[c x^n]) - \frac{3 b n (1 - e x) \log[1 - e x]}{e} + \\ & \frac{(1 - e x) (a + b \log[c x^n]) \log[1 - e x]}{e} + \frac{b n \operatorname{PolyLog}[2, e x]}{e} + 2 b n x \operatorname{PolyLog}[2, e x] - \\ & x (a + b \log[c x^n]) \operatorname{PolyLog}[2, e x] - b n x \operatorname{PolyLog}[3, e x] + x (a + b \log[c x^n]) \operatorname{PolyLog}[3, e x] \end{aligned}$$

Result (type 8, 18 leaves):

$$\int (a + b \log[c x^n]) \operatorname{PolyLog}[3, e x] dx$$

### Problem 217: Unable to integrate problem.

$$\int \frac{(a + b \log[c x^n]) \operatorname{PolyLog}[3, e x]}{x} dx$$

Optimal (type 4, 26 leaves, 2 steps):

$$(a + b \log[c x^n]) \operatorname{PolyLog}[4, e x] - b n \operatorname{PolyLog}[5, e x]$$

Result (type 8, 21 leaves):

$$\int \frac{(a + b \log[c x^n]) \operatorname{PolyLog}[3, e x]}{x} dx$$

### Problem 218: Unable to integrate problem.

$$\int \frac{(a + b \log[c x^n]) \operatorname{PolyLog}[3, e x]}{x^2} dx$$

Optimal (type 4, 174 leaves, 19 steps):

$$\begin{aligned} & 3 b n \log[x] - \frac{1}{2} b n \log[x]^2 + e \log[x] (a + b \log[c x^n]) - \\ & 3 b n \log[1 - e x] + \frac{3 b n \log[1 - e x]}{x} - e (a + b \log[c x^n]) \log[1 - e x] + \\ & \frac{(a + b \log[c x^n]) \log[1 - e x]}{x} - b n \operatorname{PolyLog}[2, e x] - \frac{2 b n \operatorname{PolyLog}[2, e x]}{x} - \\ & \frac{(a + b \log[c x^n]) \operatorname{PolyLog}[2, e x]}{x} - \frac{b n \operatorname{PolyLog}[3, e x]}{x} - \frac{(a + b \log[c x^n]) \operatorname{PolyLog}[3, e x]}{x} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{(a + b \log[c x^n]) \operatorname{PolyLog}[3, e x]}{x^2} dx$$

### Problem 219: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x]}{x^3} dx$$

Optimal (type 4, 238 leaves, 16 steps):

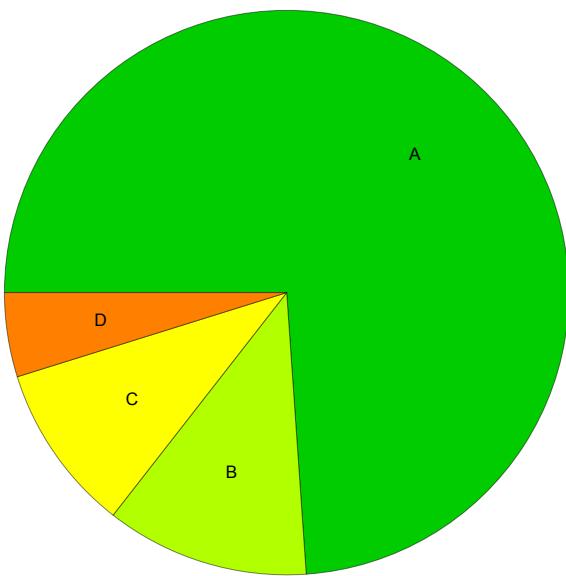
$$\begin{aligned} & -\frac{5 b e n}{16 x} + \frac{3}{16} b e^2 n \operatorname{Log}[x] - \frac{1}{16} b e^2 n \operatorname{Log}[x]^2 - \frac{e (a + b \operatorname{Log}[c x^n])}{8 x} + \frac{1}{8} e^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n]) - \\ & \frac{3}{16} b e^2 n \operatorname{Log}[1 - e x] + \frac{3 b n \operatorname{Log}[1 - e x]}{16 x^2} - \frac{1}{8} e^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 - e x] + \\ & \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 - e x]}{8 x^2} - \frac{1}{8} b e^2 n \operatorname{PolyLog}[2, e x] - \frac{b n \operatorname{PolyLog}[2, e x]}{4 x^2} - \\ & \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, e x]}{4 x^2} - \frac{b n \operatorname{PolyLog}[3, e x]}{4 x^2} - \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x]}{2 x^2} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, e x]}{x^3} dx$$

## Summary of Integration Test Results

249 integration problems



A - 184 optimal antiderivatives

B - 29 more than twice size of optimal antiderivatives

C - 24 unnecessarily complex antiderivatives

D - 12 unable to integrate problems

E - 0 integration timeouts